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An econometric analysis of benefits to tenants in alternative federal housing programs

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An econometric analysis of benefits to tenants in alternative federal housing programs

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CHAPTER I: INTRODUCTION

The basic task of this research is to estimate the direct benefits received by tenants in public housing projects, the direct benefits received by these tenants if they were to participate in housing projects operated under Section 236 of the United States Housing Act of 1968 (hereafter referred to as Section 236 housing), and the direct benefits these same tenants would receive if they were to participate in both Section 236 housing and the Rent Supplement Program (hereafter referred to as Section 236 housing with Rent Supplement). Peripherally, we obtain estimates of the income elasticity of low income families.

Chapter II contains a brief discussion of the complexities from which this study abstracts. Chapter III is an overview of the programs under study and a statement of the questions we are asking. Chapter IV tackles the question of a "best" measure of benefits. Chapter V is a short discussion of the functional forms we select for our utility functions. Chapter VI sets forth the stochastic framework for our estimation; this involves the interesting questions of estimating utility functions and properly specifying the consumer's allocation problem. In Chapter VII we discuss our data, its limitations, and identify the statistical techniques required by the data. Chapter VIII more carefully specifies the estimation techniques, outlining the random regression coefficients framework and explaining the quadratic programming approach to applying inequality constraints on parameters.

Chapter IX is a straightforward presentation of our numerical results with a limited amount of interpretation. Chapter X attempts to clarify the meaning of the numbers given in the previous chapter and contains our conclusions. Chapter XI indicates avenues of possible further study based on this research.

The results of this research are interesting, but suffer certain limitations. The estimates of tenant benefits imply society must derive considerable utility from imposing transfers in-kind vis-a-vis income transfers, indirect benefits be relatively high, supply considerations be of great import, or the programs are a very inefficient allocation of resources. Peripherally, we estimate housing demand elasticities for several price and income levels; our results contrast with aggregate estimates found in the literature. As the Department of Housing and Urban Development is using these aggregate figures in several studies of low-income households, the apparent conflict may be of some import for public policy.

The most severe restriction on the results is that estimates are not consistent due to errors in variables.

Other difficulties are the difficulty of measuring the market value of subsidized units and the lack of theoretical insights into an appropriate functional form for the utility function. Fortunately, neither alternative estimates of market values nor changes in functional form severely affect our major conclusions; however, such matters do affect interpretations of some issues, such as the distribution of benefits among tenants.

CHAPTER II: COMPLEXITIES FOREGONE

Our empirical analysis will be cast in terms of the classical marginal utility analysis of a two commodity world. However, before we descend to this extreme simplification, we will examine some of the complexities we wish to escape.

While some consumer allocation issues might be quite amenable to elementary marginal analysis, for instance the choice between scotch and bourbon, the choice of habitat is not such an issue. There are five considerations which complicate the analysis of this choice; they are: temporality, imperfect knowledge, uncertainty, transactions costs, and the discreteness of commodities.

These factors interact and have a degree of importance in this choice as in no other save, perhaps, the choices of marital status and occupation. The complexity can be highlighted by looking at the frequently made assumption that consumption and savings are separable decisions. Clearly, the choice between owning and renting involves a decision about the allocation of income over time and the assumption is unwarranted.

A complete theory of consumer choice is not our object here. However, we will look in brief how each of the five factors above relates to the choice of habitat and also how they interact. We will discuss how some of these factors might be incorporated into a study such as this one when we outline matters for further research.

The chief direct impact of temporality is on the choice to buy or rent. Ownership generally involves greater costs in the present in return for lowered costs in the future. Most home-buyers justify the expense as an "investment", that is, a form of saving. Temporarily also introduces all possibilities of change: in income, tastes, opportunities, prices, etc. Very importantly, the subjective value of a specific abode often rises with time as ties are made in the neighborhood.

Imperfect knowledge's chief impacts are dynamic effects produced through learning (as when one moves soon after occupancy because of the noisy neighbors) and the introduction of information costs (such as brokers' fees). Information costs lead one to restrict his universe of choice to some subset of the available alternatives.

Uncertainty is a highly important factor whose influence is mainly felt in conjunction with temporality and/or imperfect knowledge. One might buy out of fear of a low income in the future, or rent out of a fear of an early demise. One might buy a less attractive tract home because it is more certain to appreciate than an isolated property. A risk averter might select a home he will soon outgrow as his income rises, rather than risk being unable to afford his mortgage because of a failure to advance.

Just as transactions costs "lock" a risk averter into "too little" housing for a while, others might start out with "too much" housing in anticipation of income increases. Transactions costs are particularly high when one changes the pattern of his consumption; this will entail search costs, transportation costs, perhaps new financing costs, while the status quo costs only the 8¢ stamp on the rent or mortgage payment.

Of great significance in choosing a habitat is the discreteness of the commodities one has to choose from. While neighborhoods may vary almost continuously in quality, bedrooms, bathrooms, and the like come in integer numbers (ignoring the promoter's ½ bath); most importantly, it is very difficult to rent two-thirds of one's home and buy the rest.

In short, the choice of habitat is a matter of maximizing intertemporal utility with information and transactions costs included under conditions of imperfect knowledge and uncertainty subject to an uncertain income stream. Ow.

In this study, we reduce the vector of commodities and services making up habitat to a single, composite commodity, housing services. Moreover, we ignore temporality; nominal income in the observed year is assumed to be the relevant decision variable, not some "permanent" income. Imperfect knowledge, uncertainty, transactions costs, and discreteness are entirely ignored. Ignoring the last of these is somewhat justified in that the vast majority of public housing tenants have little or no chance of owning their own homes, the primary issue arising from discreteness; the other factors are ignored because we have no way of quantitatively dealing with them within the confines of our data.

Some of these issues are of considerable importance when considering the costs and benefits involved in relocating households, particularly length of residence. This issue in particular will be reconsidered in the section on suggestions for further reference.

CHAPTER III: THE PROGRAMS

Section 236 is relatively straightforward, in contrast to the complex of possibilities embraced in public housing. Therefore we shall first present the relevant details of 236.

Section 236 proposes to stimulate production of housing for persons of low and moderate income by offering to pay the difference between the actual monies due for principal, interest, and insurance on an FHA mortgage and the monies due if the interest were one per centum per annum. This subsidy is available if the owner of the housing project is a non-profit, limited dividend, or cooperative entity, and only so long as the project is occupied by persons of low or moderate income. The value of housing is limited according to the size of family the units can accommodate: The upper income limit for tenants is set as 135% of the public housing income limits in the area.

This last point is indicative of a fundamental difference between public housing and section 236; the latter is intended to provide for <u>both</u> moderate and low income persons, while the former serves only those of low income. The focal point of this study in terms of policy considerations is whether, in fact, section 236 <u>can</u> serve persons of low income.

A final point of great importance for our study is that tenants in section 236 housing pay as rent the greater of two sums: first, twenty-five per cent of their income (not to exceed a pre-determined "fair market rent", of which we shall say more later); or second, a rent sufficient to meet maintenance costs and the fixed expenses of the owner based on a one per cent mortgage. The import of this is tenants may pay rents in excess of twenty-five per cent of their income.

In short, section 236 offers a direct subsidy to owners of housing projects on the condition that this subsidy be passed on to the tenants who must be of low or moderate income. This subsidy is only a mortgage reduction payment and bears no relation to operating costs.

Public housing is a much more involved program. First we will outline the primary characteristics of the instrument, and then we shall mention some of the subsidiary aspects of the program. Fortunately public housing is long established and many of its possibilities have been little used, so we can focus on the more significant procedures.

Public housing stimulates the production of housing for persons of low income by offering various subsidies to local governments which undertake such projects. There are two fundamental, alternative subsidies. The first, and most common, is a fixed annual contribution paid to the local public housing agency by the Federal government and contracted for in advance. This contribution is limited to the amount necessary (as determined by HUD) to assure the low-rent character of the project, but it may not exceed a sum equal to the going Federal rate plus one per cent, times the development cost of the housing project involved. (However, additional contributions are available for units occupied by elderly persons, by persons of unusually low income, by persons displaced by government action, or by large families.) An alternative to fixed annual contributions is available; this is a capital grant large enough to assure the low rent character of the project, but in no case more than twenty-five per cent of the project's development cost.

In order for a local housing agency to qualify for one of these subsidies, local governments must meet several provisos. First, in the case of fixed contributions, all involved political subdivisions must waive real and personal property taxes on the project, in lieu of these taxes the local housing agency pays up to ten per cent of its rent collections to local government. In the case of capital grants, local governments must provide contributions (in almost any form) to the project equal in value to at least twenty per cent of the development costs. Finally, the local community must contract to eliminate as many unsafe or insanitary dwellings in the community as there are new units in the housing project.

A further subsidy of considerable import is made available in the form of below market rate loans for the development of low rent projects. These loans may not exceed ninety per cent of the development cost in value, and they bear interest equal to the going Federal rate plus $\frac{1}{2}\%$.

Another direct subsidy is Tenant Service Grants which are given to support counselling and advisory programs within the projects. An indirect subsidy is made available by freeing bonds issued by local housing authorities from federal taxation; moreover, annual contributions may be used collateral for such bonds.

Income limits in public housing are set by the local authority with the approval of HUD. Rents may not exceed twenty-five per cent of the tenant's income nor may they exceed eighty per cent of the lowest local rents for unsubsidized, standard apartments. In addition to showing preference for persons of lowest income, local agencies are expected to give first priority to the elderly, veterans, large families, and persons displaced by government action.

Public housing also has strictures and policies regarding relocation of persons displaced by public housing, rehabilitation as a source of public housing units, assurance of the low-rent character of subsidized projects, foreclosure and disposal rights of the Federal government, and even terms under which public housing may be sold to its tenants. In practice relocation is the only of these of real significance, and its role as an impediment is partially reflected by the inclusion of relocation costs in total development costs.

In short, public housing involves direct and indirect subsidies to local governmental agencies which are shifted forward to the tenants of the low-rent housing. These subsidies take several forms, and since the size of annual contributions are dependent on construction costs, location, size of project, rental potential of tenants, and the like, these subsidies do not reflect the operating costs of the project.

The Rent Supplement Program subsidizes participants by paying any rent obligations in excess of twenty-five per cent of the participants' income. When coupled with Section 236 housing there is the further proviso that the participant must pay at least thirty per cent of the standard minimum rent for his unit. In other contexts, the Rent Supplement Program imposes a ceiling on the market rent of units embraced by the program, but this does not concern us here.

CHAPTER IV: MEASURES OF BENEFITS

Our first task is to settle on a best measure of direct tenant benefits. Naive discussion and the existent literature suggest several measures for consideration. For enlightment on these issues we rely heavily on the careful, insightful analysis of Sir John Hicks as presented in his Revision of Demand Theory (15).

Housing programs generally yield two distinct kinds of benefits. First, participants are made better off because the programs allow the purchase of housing services at prices below the market price of housing; the benefits from this price subsidy are the "direct" benefits of interest in this study. Second is a class of less easily defined benefits which accrue to society as a whole. Some elements of this second class accrue from the altruistic preferences of the general populúlation; people are, in general, made better off when the lot of less fortunate members of society is improved. Other elements of this class are due to the general population's desire to alter the composition of the poor's consumption; housing programs cause participants to consume more housing than they would otherwise (and even more than they might consume under some cash grant programs). Still other elements of this class are due to externalities; public housing may generate neighborhood effects, or may make better citizens of its tenants, and these effects redound to the benefit of the whole population.

We leave to others the difficult task of measuring these latter, indirect benefits; our concern is with the direct benefits to tenants. Our results, however, are not without interest to persons concerned with indirect benefits. We presume society is both altruistic and interested in altering the composition of consumption of its intended beneficiaries; this study will identify the extent to which programs improve the lot of participants and alter their consumption patterns, as well as distinguishing the cost of the former from that of the latter. Indeed, it is these concerns which enable us to settle on a "best" measure.

A naive measure of the direct tenant benefits would be the difference between the tenants' pre-participation rent and his program rent. While it is true that most subsidized programs do involve a decrease in tenant rents, thus allowing participants to purchase more of other goods, this is a poor measure as it takes no account of the change in the quantity of housing services consumed by the tenant. It is entirely possible that a program might involve rents higher than those the participants paid outside the program and still benefit the tenants positively; the added quantity of housing obtained at the higher, subsidized rent might easily compensate for the decreased consumption of other goods. Who would not accept the finest house in town for a rent one dollar higher than they are currently paying? Probably only the family already in that house.

Prescott (21) and Bish (2) have used the difference between the market rent of the subsidized unit and the subsidized rent of the unit to estimate benefits. This measure can be viewed as an approximation of the Marshallian consumer's surplus measure used by Prescott and Olsen (20); in another paper (19) Olsen suggests the Marshallian measure, but estimates it with an approximation.

DeSalvo (11) and Aaron and Furstenberg (1) have suggested still another measure, called the equivalent variation. We use this measure here.

The remainder of this chapter compares these measures found in the literature and discusses both their relationships, one to another, and our reasons for selecting the equivalent variation as the best measure for our purposes.

Here, and throughout this study, we postulate a two commodity world composed of housing services (a bundle embracing many elements, including utilities), and a composite commodity make up of all other goods and services. We will always denote the quantity of housing services by H and the quantity of the composite commodity by E; specific quantities will be differentiated by superscripts, such as with H^m and H^s. The prices of H and E will be designated P_H and P_E throughout; specific prices will be differentiated by superscript, such as P_H^m and P_H^s. Income is designated Y.

The greatest constraint imposed by this framework is that it requires the relative prices of goods within each commodity group to be constant. Such an assumption is common and generally necessary to reduce issues to manageable proportions.

We note here that "housing services" excludes furniture. This could present problems in assessing market rent data if we had reason to believe some observations were on furnished apartments. However, subsidized units are unfurnished and it would seem a prohibitive burden on a low income family to move in without furniture and have to obtain it on short order. For this reason, we expect few if any of our observations to be on furnished units.

Marshall defined consumer's surplus as the excess of the price which he (the consumer) would be willing to expend rather than go without the thing, over what he actually does pay (17). Marshall pointed out that this excess would equal the area under the demand curve to the left of the quantity purchased less the payment for the commodity, subject to the important assumption that the marginal utility of money be constant. Thus in Fig. 1, if P_H^m is the market price of housing, H^m the quantity to be bought at that price, and D the demand function giving P_H as a function of H, then the consumer's surplus for this buyer of housing in the market place is:

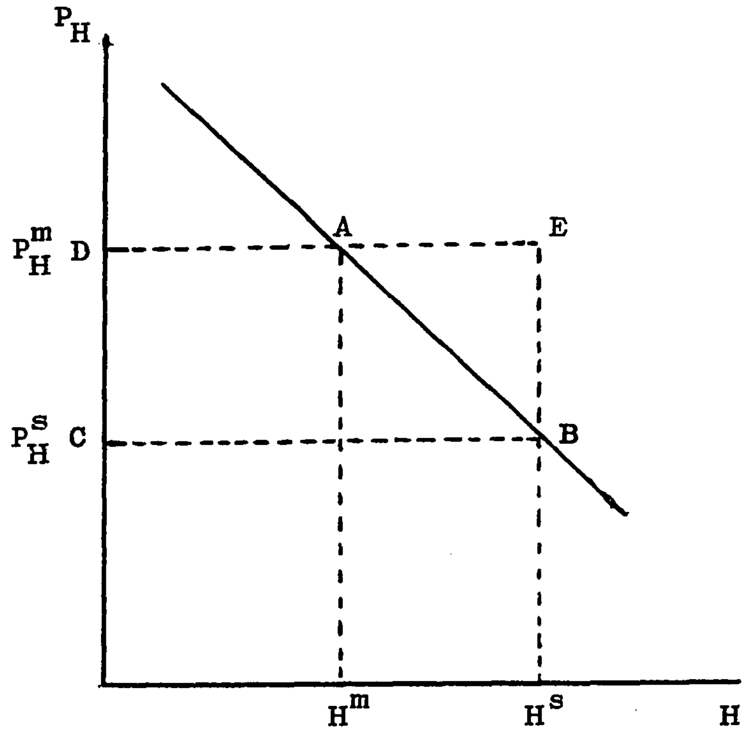

Figure 1. Marshallian Consumer's Surplus

$$\int_{o}^{H^m} D \, dM - P_H^m M^m$$

presuming the marginal utility of money is constant. To measure the benefits to this tenant from a subsidized housing program in which he buys H^s at a price P_H^s, we simply take the change in consumer's surplus from the market position to the subsidized position; that is:

$$\text{Benefit} = \int_{H^m}^{H^s} D \, dH + P_H^m H^m - P_H^s H^s.$$

This is the area A B C D in Fig. 1.

Clearly, the market rent of the subsidized unit would be P_H^m times H^s; the difference between this and the subsidized rent is B C D E. Thus if demand were perfectly elastic with respect to price, the market less subsidized rent measure of benefits would equal the Marshallian measure. The more elastic the demand for housing services with respect to price, the poorer this measure as an approximation to Marshall's measure.

The housing programs under study do not generally allow participants to pick whatever amount of housing they want at the subsidized price. In order to alter the pattern of consumption of participants, the programs offer a specific quantity of housing services (based on family composition) at a given price (based on income). Thus there is no assurance that H^s will be a point on the tenant's demand curve. In practice this offers no difficulty, such a case is depicted in Fig. 2, where prices and quantities are to be interpreted as in Fig. 1. In Fig. 2 the benefits are measured by A B C D E. While there are no practical problems here, there are theoretical difficulties, as we show later.

Marshall's measure, then, purports to be the amount of money we could take from a person to leave him no better off than he was prior to entering the program. This measure is, in itself, uninteresting from a policy point of view because there is no interest in actually "taxing" participants in this fashion. However, the Marshallian measure would be of interest if it offered a reliable, simple approximation of a more policy oriented benefit measure. We now turn to such a policy oriented measure and investigate its relation to the Marshallian measure.

Hick's equivalent variation measure of benefits is suggested for housing analysis in De Salvo (11) and Aaron and Furstenberg (1). The measure is the change in income which would make the participant just as well as the proposed program. This measure is ideally suited for our purposes. It tells us in an untuitively appealing way how much better off participants in housing programs are made ("It is like increasing their incomes x dollars."); it tells us how much it would cost to improve the lot of tenants just as much as the program does, if we were to forego affecting their

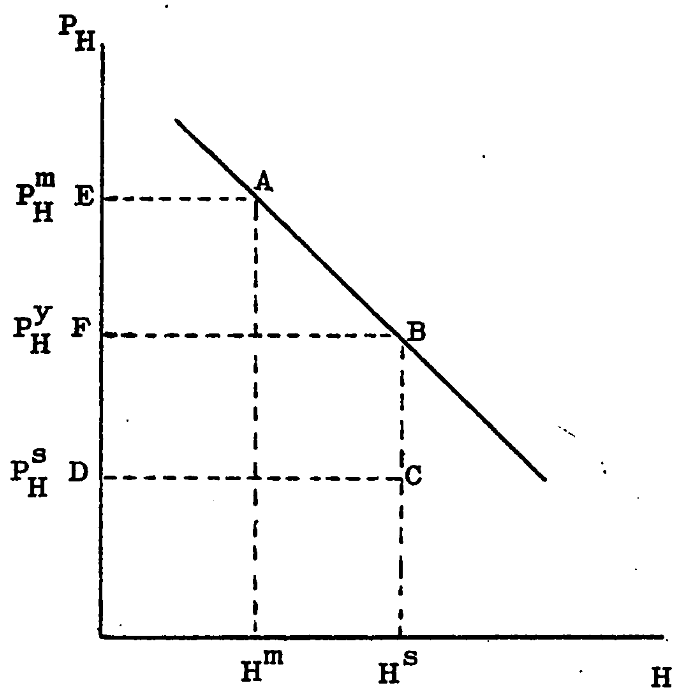

Figure 2. Marshall's Measure and Points off the Demand Curve

consumption pattern (that is, if we let them do as they please at market prices rather than assuring their consumption of some one commodity rises); finally, it allows us to compare the participants' actual consumption of housing with what they would consume if given an equivalent cash grant to see how much consumption patterns are altered. Moreover, the equivalent measure is much more robust than Marshall's measure in cases off the demand curve.

The following comparison of Marshall's measure with that of Hicks, is derived from Chapter X of Hicks' Revision of Demand Theory (15). Hicks discussion is in terms of the "compensating variation" and we have transformed the arguments appropriately. The points made about points off the demand curve are not from Hicks, and we believe they are new.

For simplicity we begin with the case in which the consumer is subjected to no constraints other than his money income and either market or subsidized prices. Figure 3 shows such a case for subsidized housing prices.

The line YY is the budget line corresponding to initial money income and market prices. YY' is the budget line corresponding to initial money income and subsidized prices; the subsidized price of housing, P_H^s, is below the market price, P_H^m.

At initial prices, the consumer maximizes utility at A on indifference curve U_1, at subsidized prices the consumer

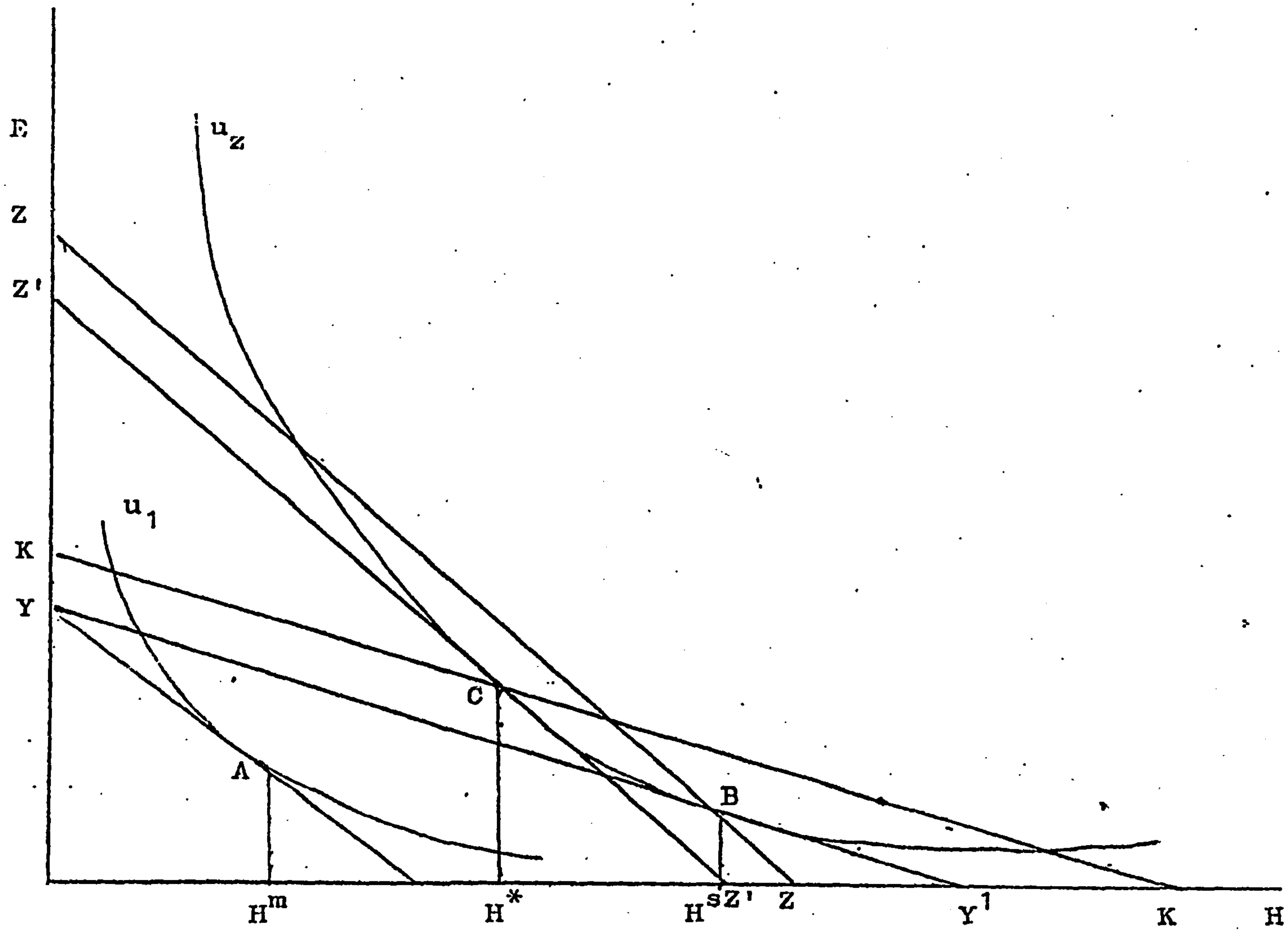

Figure 3. Unconstrained Housing Price Subsidy

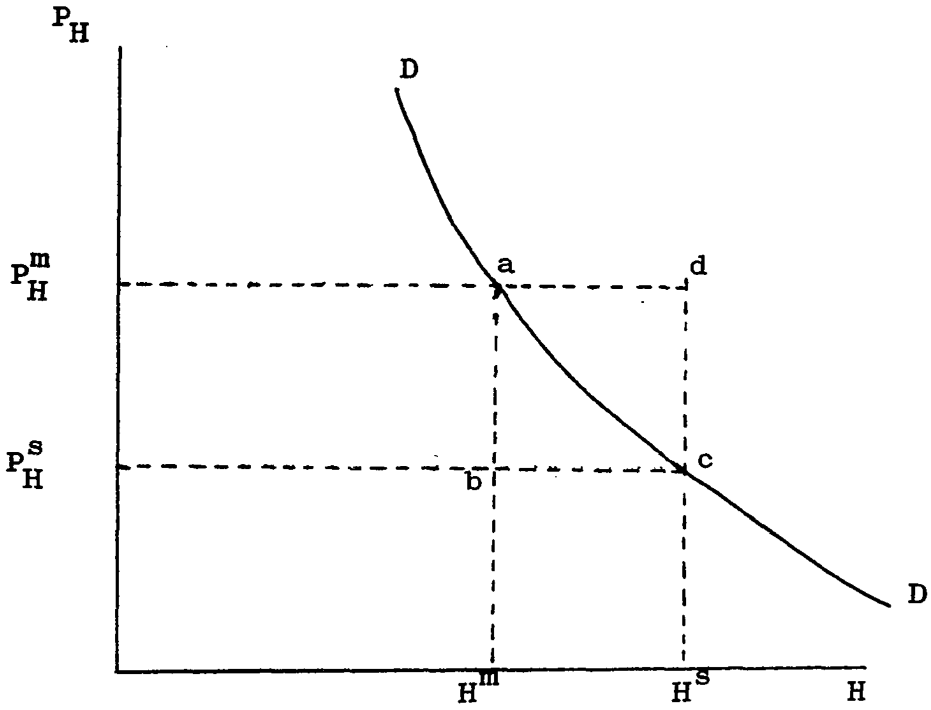

Figure 4. The Bounds of the Equivalent Measure

maximizes utility at B on the higher indifference curve U_2. The line ZZ is the budget line which would just allow the consumer to buy the bundle B at market prices. Z^1Z^1 is the budget line corresponding to the money income which would just allow the consumer, at market prices, to achieve the same level of satisfaction attainable under the price subsidy. The difference in money incomes between YY (or YY') and Z^1Z^1 is the equivalent measure of benefits.

Note that the money income corresponding to ZZ is necessarily greater than that corresponding to Z^1Z^1.

In Figure 4, H^* is the amount of housing services which the consumer would obtain if he were given an equivalent cash grant in lieu of a price subsidy; H^s is the amount of housing services which the consumer would choose under the price subsidy. P_H^m is the market price of housing services and P_H^s is the subsidized price.

The bundle of goods B in Figure 3, can be bought with either the initial income at subsidized prices or with the income corresponding to ZZ at market prices. Since in either case the same amount is spent on goods other than housing (P_E is constant), the difference in these two incomes is the difference in the valuation of H^s at market and subsidized prices, $(P_H^m - P_H^s)\ H^s$. This difference is necessarily greater than the equivalent measure, and is represented by the area P_H^m d c P_H^s in Figure 4.

Now suppose that the equivalent variation in income has been granted and market prices maintained. In such a case, the consumer is at point a in Figure 4. Now consider a budget line K K, through c in Figure 3 corresponding to subsidized prices (and this parallel and to the right of YY'). This budget line corresponds to a money income less than that of Z'Z' but greater than that of YY; the difference between this money income and that of Z'Z' is $(P_H^m - P_H^s)\ H^E$. This last difference is represented in Figure 4 by the area P_H^m a b P_H^s; but if the money income of KK lies between those of Z'Z' and YY, this difference must be <u>less</u> than the equivalent measure. Thus we have upper and lower bounds on the equivalent measure.

Now let us consider the points a and c in Figure 4. They are, in fact, two points on a compensated demand curve, DD. Moreover, if we view the price change from P_H^m to P_H^s as made up of an infinite series of small price changes, the bounds of the equivalent measure converge to DD. Thus the equivalent measure of benefits for unconstrained price subsidies is the area P_H^m a c P_H^s for the compensated demand curve through the subsidized position. Now we can show the relationship between Marshall's measure and the equivalent measure for points on the demand curve.

In Figure 5 we present the compensated demand curve, DD, through (P_H^s, H^s) and the Marshallian demand curve CC on the presumption H is a normal good; CC is to the left of DD for

H below H^s and to the right of DD for H above H^s.

Marshall's measure of the benefits of a fall in price from P_H^m to P_H^s is the area P_H^m XZ P_H^s. This is XYZ less than the equivalent measure. Thus, for normal goods, Marshall's measure is an underestimator of the equivalent measure. For inferior goods Marshall's measure would be an overestimator of the equivalent measure. All this is subject, of course, to the requirement that (H^s, P_H^s) be on the consumer's demand curves. (In the odd case that there is no income effect, Marshall's measure coincides with the equivalent measure.)

What becomes of this regular relationship between Marshall's measure and the equivalent measure if we abandon the assumption of unconstrained subsidies? Suppose (H^s, P_H^s) is below the demand curve as in Figure 2. Marshall's measure, in this case, is equal to the benefits from a price fall to P_H^x plus the area fbcd. Let us consider this subsidy as having two parts, first, a price fall to P_H^x and second, a constrained income grant of fbcd (spend it on anything except housing).

For the first part, Marshall's measure underestimates the equivalent measure (if H is a normal good); but for the second part, Marshall's measure overestimates the equivalent measure, because unless I do not wish to spend any added income on H, an unconstrained grant of less than fbcd will improve my lot by as much as a constrained increase of fbcd.

Thus, for constrained subsidies of normal goods, there

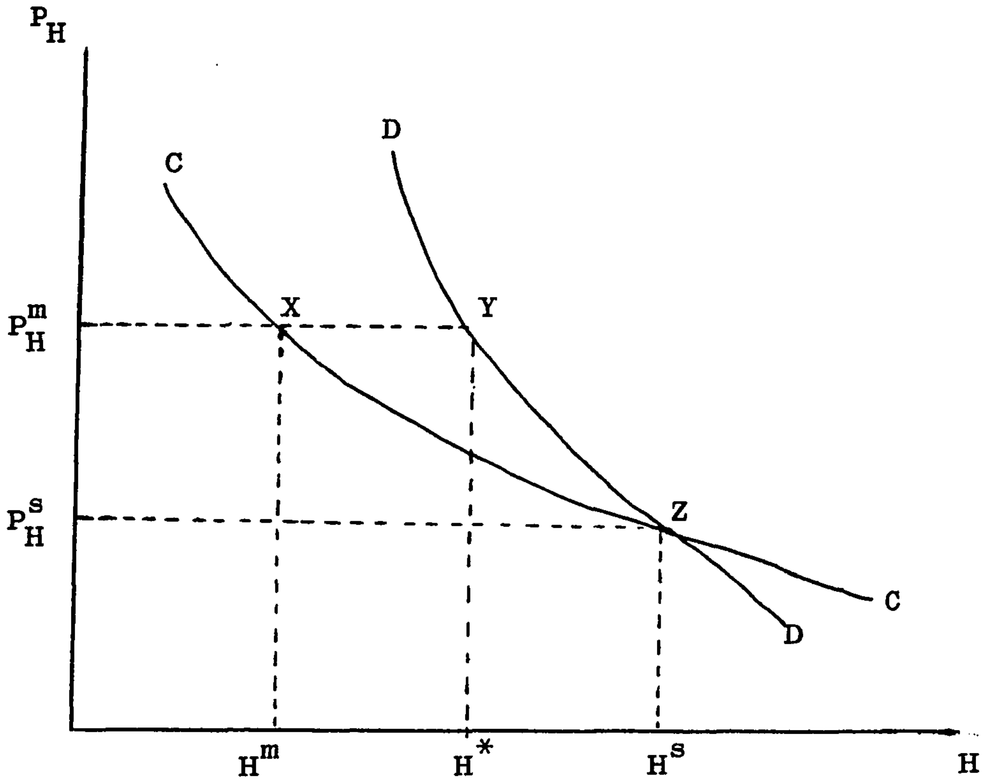

Figure 5. Marshall's Measure and the Equivalent Measure

is no general, certain relationship between Marshall's measure and the equivalent measure. Since we are dealing with constrained subsidies of housing, which is almost certainly a normal good, Marshall's measure is of little use as an approximation of the equivalent measure. Therefore, despite the greater simplicity in application of Marshall's measure, we feel it is necessary to attempt a direct measurement of the equivalent variation.

We should note in passing, that if one seeks to estimate the equivalent measure from the compensated demand curve (presuming it, or a suitable approximation, were available), the same problems with regard to points off the demand curve would arise as with the Marshallian curve. For this reason, we must proceed to obtain an estimate of the consumers indifference map or, equivalently, of his marginal rate of substitution as a function of H and E.

An analytic expression for the equivalent measure as a function of prices, income, and subsidized quantities is not necessarily possible for every valid utility function. Fortunately, iterative computer programs can generate approximations to any desired degree of accuracy.

The computer program utilized here uses an estimated utility function to identify the indifference map on which the subsidized bundle lies and proceeds to search for the income which would just enable the consumer to attain that same indifference at market prices. The difference between actual income and the income found above is the equivalent measure.

Our tasks in this study follow directly from these considerations. We must first confront the task of selecting a utility function (or several); next we must formulate a proper stochastic model to estimate the parameters of this function, and then we must use our estimates to approximate estimates of benefits based on our utility function parameter estimates. These efforts are taken up in the following chapters.

In summary, we have settled on Hick's equivalent variation as our measure of benefits. Thus we are implicitly comparing programs to the alternative of an income maintenence plan and asking what income subsidy would make tenants feel just as well off as the program does. We reject Marshall's measure as a useful approximation chiefly because of that measures inaccuracy in dealing with points off the demand curve.

CHAPTER V: FUNCTIONAL FORMS

If we are to estimate the equivalent measure of direct tenant benefits for public housing and section 236 housing we must first find an appropriate form for the utility function. De Salvo (11) and Aaron and Furstenberg (1) rely on price and unitary income elasticity estimates found elsewhere in the literature to suggest using constant elasticity of substitution (CES) or Cobb-Douglas utility functions. There are two problems with this approach.

First, elasticity estimates found in the literature are for *aggregate* demand functions, and thus have nothing to say, in general, about the parameters of individual utility functions. Indeed, the one disaggregated study in the literature, by Lee (16), estimated income elasticities of much less than one.

Second, the demand functions used in previous work are not generally consistent with classical utility functions. For example, a constant income elasticity of greater than one will imply that the weighted sum of income elasticities, where the weights are budget shares, exceeds unity as income rises, in violation of the Engel aggregation condition. This is no indictment of these studies, as aggregate curves need not satisfy the classical conditions; however, it does highlight their inappropriateness for our purposes.

Theory sheds little light on the choice of a utility function. Most efforts to develop useful functional forms have involved searches for manageable traits such as linearity and additivity for use in demand theory (13) or takeovers from production theory. The Cobb-Douglas form and the CES form are of the latter type.

Due to its simplicity, the Cobb-Douglas is an attractive specification. Indeed, we use it in this study for this very reason. However, this simplicity implies strong restrictions, namely the a priori assertion that price and income elasticities are both unitary.

The CES specification escapes the unitary price elasticity, but maintains a unitary income elasticity. In order to relax this last restriction, we utilize a generalization of the CES which will allow us to test the sensitivity of our results to specification changes and, additionally, permit a test of the unitary income elasticity hypothesis, as the CES function is a special case of the generalized CES.

Thus, the two functional forms used in this study are the Cobb-Douglas:

$$U = H^a E^b$$

and the generalized CES:

$$U = (kH^c + E^d)^m$$

where a, b, c, d, k, and m are parameters and U refers to the level of utility. Clearly, $c = d = 1/m$ reduces the latter form to the CES.

The theory of consumer behavior requires the utility function to be quasi-concave, at least in the relevant region. For the Cobb-Douglas, this implies a and b must both be greater than zero. For the generalized CES the implication is that r must be greater than zero while c, d, and m share the same sign; for convenience we assume this sign is positive. Moreover, for the generalized CES, c and d must be less than one unless H and E are appropriately bounded.

These *a priori* theoretical constraints can be used in either of two ways. First, they can be used as additional information in the regression in an effort to obtain more efficient answers; second, the constraints can be used to test hypothesized models. In the light of the data restrictions we face, we use the *a priori* theoretical constraints as an integral part of our specification of the model. The consequences of this for parameter estimation are discussed in Chapter VII.

Before passing on to the question of a stochastic specification of our problem, we should point out the income elasticity of housing demand associated with the generalized CES; it is:

$$Y/(P_H H + \left(\frac{c-1}{d-1}\right) P_E E).$$

Note if c equals d, the income elasticity is unitary; thus testing the equality of c and d is a test of the unitary income elasticity hypothesis.

In the next chapter we present the formal stochastic specifications used in estimating the Cobb-Douglas and generalized CES utility functions.

CHAPTER VI: THE STOCHASTIC MODELS

A correct specification of the consumer's allocation problem is not an easy task, even if we take our two commodity world and a particular form of the utility function as given. However, the starting place is not hard to imagine. One begins with the classical maximization condition that the ratio of marginal utilities is equated to the ratio of prices, subject to the budget constraint.

For the Cobb-Douglas function this leaves:

$$\frac{P_H}{P_R} = \frac{a^E}{b^H}. \qquad 1$$

For the generalized CES, on the other hand, we have:

$$\frac{P_H}{P_E} = \frac{kc\ H^{c-1}}{d\ E^{d-1}} \qquad 2$$

While in both cases, we must add:

$$Y = P_H H + P_E E.$$

Note that if the exponents in Equation 2 are both -1, the generalized CES reduces to the Cobb-Douglas case. Estimation of the generalized CES, then, allows a test of the hypothesis that the Cobb-Douglas is the appropriate form.

We will continue using both formulations for two reasons. First, it is of interest to see how the Cobb-Douglas performs as an approximation of the generalized CES. The Cobb-Douglas is easier to estimate and permits an analytical form of the equivalent measure of benefits:

$$B = \left(\frac{R_m}{a}\right)^a \left(\frac{y_o - R_s}{b}\right)^b - y \qquad 3$$

where y_o is initial income, R_m the market rent of the subsidized unit, R_s the subsidized rent, and B benefits. It would be helpful if the Cobb-Douglas offered close approximation, even under conditions in which we reject the hypothesis that the Cobb-Douglas were the true functional form.

Our second, less compelling, reason for continuing with both forms is that some researchers seem to have strong a priori beliefs about the appropriateness of the Cobb-Douglas hypothesis in a Bayesian sense. While we do not share these strong priors, we felt others interested in this research might.

We have not yet made the framework stochastic. In doing so we will also confront the simultaneous nature of the allocation process.

The consumer's decision variables are not, in fact, marginal utilities; nor is the decision variable the ratio of two such items.

Taking account of the stochastic element, the simultaneous equation model for the Cobb-Douglas function would be:

$$E = \frac{b}{a}\,\frac{P_H H}{P_E} + u$$

$$H = \frac{a}{b}\,\frac{P_E E}{P_H} + e$$

$$Y = P_H H + P_E E.$$

Similarly, for the generalized CES we have; in logs:

$$\ln E = \frac{1}{d-1}\ln\left(\frac{ck}{d}\right) - \frac{1}{d-1}\ln\left(\frac{P_H}{P_E}\right) + \frac{c-1}{d-1}\ln u + \ln u \qquad 4$$

$$\ln H = \frac{1}{1-c}\ln\left(\frac{ck}{d}\right) - \frac{1}{1-c}\ln\left(\frac{P_H}{P_E}\right) + \frac{d-1}{c-1}\ln E + \ln e \qquad 5$$

$$Y = P_H H + P_E E$$

The first point to note is that neither of these specifications can yield all the parameters of our utility functions. The parameter m does not even appear in the latter system, and the former system can only yield an estimate of the ratio of a to b, not particular values for each. However, this should come as no surprise; it is only a reflection of the arbitrary nature of the utility function. Subject to the concavity conditions, we can select any values for a and b consistent with the estimated ration, and any value at all for m. This feature will be the same in all specifications of the problem.

The second point to note is that certain non-linear relations hold between parameters in different equations. For purposes of estimation, the two specifications above would have to be rewritten as follows:

$$E = a_E \frac{P_H P}{P_E} + u$$

$$H = a_H \frac{P_E E}{P_H} + e$$

$$Y = P_H H + P_E E \qquad 6$$

$$a_E = {}^1/_{a_H} \qquad 7$$

for the Cobb-Douglas and:

$$\ln E = a_E + B_E \ln \left(\frac{P_H}{P_E}\right) + W_E \ln^H + \mathrm{lu}\ u \qquad 8$$

$$\ln H = a_H + B_H \ln \left(\frac{P_H}{P_E}\right) + W_H \ln E + \mathrm{lu}\ e \qquad 9$$

$$Y = P_H H + P_E E \qquad 10$$

$$\frac{a_E}{a_H} = \frac{B_E}{B_M} \qquad 11$$

$$W_E = 1/W_H \qquad 12$$

$$W_E = \frac{a_E}{a_H}$$

for the generalized CES.

We now go about imposing the parameter constraints. In the case of the Cobb-Douglas function, this is not difficult.

Equation 6 implies

$$P_H H = Y - P_E E$$

and

$$P_E E = Y - P_H H$$

Thus Equations 4 and 5 can be rewritten as

$$E = a_E \frac{Y - P_E E}{P_E} + u$$

$$H = a_H \frac{Y - P_H H}{P_H} + e$$

or, rearranging terms once more, as

$$E = \frac{a_E}{1+a_E} \qquad \frac{Y}{P_E} + u$$

$$H = \frac{a_H}{1+a_H} \qquad \frac{Y}{P_H} + e$$

Now it only remains to multiply through by P_E and P_m, respectively.

$$P_E E = \frac{a_E}{1+a_E} \; Y \;\; uP_E \qquad 13$$

$$P_H H = \frac{a_H}{1+a_H} \; Y + eP_H \qquad 14$$

Ordinary least squares estimation with Equations 13 and 14 will yield estimates of $^aE/1+a_E$ and $^aH/1+a_H$. Moreover, these imply estimates of a_E and a_H which meet the constraint imposed by Equation 7. Even under ideal conditions these estimates would not be unbiased, but they would have the property of consistency in such an instance. Thus the ratio a/b can be equally well estimated by applying ordinary least squares to either Equation 13 or Equation 14.

In the generalized CES case, we proceed little differently. We begin by taking anti-logs in Equations 4 and 5. This gives:

$$E = \left(\frac{ck}{d}\right)^{\frac{1}{d-1}} \left(\frac{P_H}{P_E}\right)^{\frac{1}{1-d}} (H)^{\frac{c-1}{d-1}} u \qquad 15$$

$$H = \left(\frac{ck}{d}\right)^{\frac{1}{1-c}} \left(\frac{P_H}{P_E}\right)^{\frac{1}{c-1}} (E)^{\frac{d-1}{c-1}} e \qquad 16$$

We now raise both sides of Equation 15 to the (d-1) the power and both sides of Equation 16 to the (c-1) the power.

$$E^{(d-1)} = \left(\frac{ck}{d}\right) \left(\frac{P_E}{P_H}\right)^{(c-1)} u^{(d-1)}$$

$$H^{(c-1)} = \left(\frac{d}{ck}\right) \left(\frac{P_H}{P_E}\right) E^{d-1} e^{(c-1)}$$

Dividing $E^{(d-1)}$ by $H^{(c-1)}$ and rearranging terms yields:

$$E^{2(d-1)} = \left(\frac{ck\, P_E}{dP_H}\right)^2 H^{2(c-1)} u^{(d-1)} e^{(1-c)} \qquad 17$$

If we allow $u^{\frac{1}{2}(1-c)} e^{\frac{1}{2}(1-c)}$ to be represented by w, we can take the square roots of both sides of Equation 17 and rearrange terms to obtain:

$$\frac{P_H E}{P_E H} = \frac{ck}{d} H^{(c-2)} E^{(2-d)} \qquad 18$$

Thus we can estimate the parameters of the relation

$$\ln \frac{P_H E}{P_E H} = \ln \frac{ck}{d} + (c-2) \ln H + (2-d) \ln E + \ln w$$

to obtain estimates of the parameters of the "true" system represented by Equation 8 through 12. Of course, this same specification would have followed directly if we had selected the ratio E/H as the decision variable of the household and had simply stuck a multiplicative error term onto Equation 2.

The purpose of our more tedious development is twofold. First, this approach makes the specification of the <u>system</u> the same for both particular functional forms of the utility function; we generally specify an equation for H, an equation for E, the budget constraint, and a series of parameter constraints peculiar to the underlying functional form of the utility function.

The second fruit of this roundabout approach is to highlight the arbitrariness of any assumptions about the distribution lnw. Assuming lnw to be normally distributed would imply peculiar distributions for lnu and ln e of earlier equations, and vice-versa. The implication is that strict attention to t tables and F statistics could prove very misleading.

One important point is to note that Equation 18 is not amenable to classical least squares estimation; ln H and ln E are both correlated with the error term. To deal with these facts, we use the method of instrumental variables which, under ideal conditions, would yield consistent estimates. The instruments used are Y, P_H, P_H/P_E and a constant term; the difference here between our approach and that of two stage least squares is that P_H does not formally appear in the model as an exogeneous variable.

Thus if conditions were ideal, we could obtain consistent estimates of the parameters of both the Cobb-Douglas and the generalized CES utility functions (or actually of the indifference maps these functions imply). Unfortunately, errors in the observation of P_H and P_E imply that the estimates will not, in fact, be consistent. However, we still utilize the method of instrumental variables in the generalized CES cas on the assumption that the inconsistency due to errors in variables is less than that which would arise from the correlation of ln H, and ln E with ln w.

Before passing on to a discussion of our data we will point up a further estimation problem. In Chapter IV we remarked that our benefit measure will be a function, perhaps a non-analytic function in the parameters of the utility function. To avoid the task of finding an analytic function for the generalized CES benefit measure, we use an iterative computer program to generate approximations to within one dollar per year. In this program we use the parameter estimates for the Cobb-Douglas and generalized CES based on our public housing sample. The fact that our real interest is in the benefit measure minimizes the importance of unbiased parameter estimates for the utility functions--the characteristic would not generally carry over; on the other hand, consistency becomes more important as it is a property which is preserved through functional transformations.

It is important to note that as a program becomes more and more efficient, a bias in the estimate of benefits will grow in importance. As pointed out in Chapter IV, there is an upper bound on the benefits measured for any transfer in kind, while there is, in principle, no lower bound. As the true benefits approach the upper bound, observations in excess of the true value become less and less likely.

It follows that as estimated benefits grow closer to the maximum level, we must be more stringent in specifying acceptable standard errors when judging the inefficiency of the program in question.

CHAPTER VII: THE DATA

The bulk of our data is from public housing and Section 236 housing application forms. These forms provide family composition, race and age of household head, family income, subsidized rent, previous rent (in the case of public housing), and the unsubsidized cost of the apartment (in the case of 236 housing). Income and the previous market rent of public housing tenants is crucial to estimating the indifference relations; these data indicate actual market behavior.

Relative prices of housing and other goods, inter-city price indices, and quantity indices are derived from the Bureau of Labor Statistics' "Three Budgets for an Urban Family of Four" (4, 5, 6, 7, 8). These budgets, which are compiled semi-annually, give the costs of standardized bundles of commodities (including housing services) in each of cities. There are separate sets of bundles for each of three income classes; since our study is limited to persons of low income, we use the standardized bundles associated with the lowest cost budget. These estimates of prices must be viewed as random variables and are the chief source of inconsistency in our estimates.

Information on the market value of public housing units is obtained in two ways. One is to use the ceiling rents set by local housing authorities; this information was included in reports sent to the Department of Housing and Urban Development. The second method is to use a regression technique used by Olsen (18); the requisite characteristics were obtained from HUD reports (10).

Aside from the cross-sectional nature of the study, the crudity of the data is the strongest limitation in our analysis. The credibility of our results depends on a full understanding of the assumptions underlying the use of our data.

Family income and previous rent are data supplied by applicants to public housing. There is a definite incentive for prospective tenants to understate income. To reinforce understated income claims, applicants may also understate their previous rent; on the other hand, pride could account for some overstatement of previous rent. Of more serious concern are the families whose move to public housing is due to a sudden drop in income; in these cases previous rent has nothing to do with reported present income. Hopefully these cases serve to counterbalance under reporting of income. Also of concern are applicants who are coming from the homes or friends where their rent bore no relation to market rentals. Some applications were excluded from our study because they gave obvious evidence of these deficiencies; data were ignored if previous rent exceeded income or was equal to zero. Not all previous rents included utilities as we assume; the extent of this deficiency is unknown, but we feel most low rent units include utilities and that the bias introduced is small. In short, the rent income data are imperfect, but if this were the worst of our problems, we should indeed be glad of it.

The unsubsidized cost of the apartment should be a good measure of the market value of Section 236 units if the government is an efficient producer of housing; unfortunately it is generally agreed that the unsubsidized cost seriously overestimates the market value of Section 236 units. Recent investigations by the Department of Housing and Urban Development have found that market value is approximately ninety-one per cent of unsubsidized cost. This is the figure we use.

We obtain a quantity index for housing by assuming that a composite bundle called "housing services" exists and is appropriately represented by the bundle of services included in the BLS study. We further assume that a unit of housing services costs the same everywhere in a given market. (This assumption is weaker than that of perfect competition.) The extent to which poor blacks pay more or less than poor whites, or female headed households more or less than male headed households, these assumptions are unwarranted and may bias our result. Evidence is not strong that these differences are so strong as to invalidate our results.

On these assumptions, dividing the rent of a given unit by the rent of the BLS standardized bundle of housing yields an index of the quantity of housing services yielded by the apartment in question. The price of a unit of housing is then the rent of the standardized unit.

An index for the quantity of other goods is similarly obtained, and the price is the price of the BLS bundle of commodities other than housing.

These price and quantity measures benefit from the narrow social and economic dispersion of our families; within a narrow range of incomes expenditure should change primarily in quantities of various goods purchased more than in kind of goods purchased.

The market value of subsidized units is the single-most unreliable datum we have. In the case of Section 236 units, we use the ninety-one per cent of the cost of production included on the applications of tenants. For public housing we use several estimates, three based on local housing authority estimates and one based on a regression equation estimated by Olsen (18).

In most cities, local housing authorities set maximum, or ceiling rents for their public housing units, according to the number of bedrooms. These ceiling rents are purportedly the market values of the units, but there is no evidence that local authorities have any rigorous method for determining these values. Our technique has been to base a series of estimates (using inter-city price indices derived from the BLS statistics) on the ceilings for each city and to select the maximum, minimum, and mean values for each apartment size in each city as estimates.

An alternative to these measures is based on the regression equation of Olsen estimated in a study of New York City rent control. We selected average values for many characteristics (such as floor in dwelling occupied), but the primary variables were available. The regression equation is used to estimate the value of projects as if they were located in New York; inter-city price indices derived from the BLS statistics are then used to translate the values to correspond to the cities in which the projects are located. These values are estimated for each apartment size in each project appearing in the sample.

Unlike Section 236 units, we feel we have somewhat acceptable upper and lower bounds for the benefits to tenants insofar as provide reasonable upper and lower bounds on market values for public housing units.

Recent investigations by the Department of Housing and Urban Development indicate that the cost of public housing units is about one hundred and twenty per cent of their market value. This is the figure we use.

A final note should be made on the size of our sample. We have a total of thirteen hundred and eighty-eight observations, a considerable number. Even after stratifying the sample by family size and composition (nine classes, in all) the samples run from a smallest number of forty-nine to a largest of five hundred and fourteen.

The apartments to be hypothetically given to public housing tenants under the provisions of Section 236 are determined by calculating the average apartment value and subsidized rent for each family composition in a large sample of actual Section 236 housing tenants. This information was obtained from Section 236 applications.

CHAPTER VIII. ESTIMATION TECHNIQUES

This chapter presents the estimation techniques actually used in this study as well as a potential source of bias in the benefit estimates. Random regression coefficients and inequalities as constraints are the chief estimation issues.

We assume all households have utility functions of the same form but that the parameters vary from household to household. The distribution of parameters is assumed to have finite mean and variance.

Thus, the specifications of Chapter VI must be modified. Equation 14 becomes:

$$P_H H = \left(\frac{a_H}{1+a_H} + \varepsilon_o\right) Y + uP_H \qquad 20$$

or,

$$P_H H = \frac{d_H}{1+a_H} \; Y + (\varepsilon_o Y + u^P H) \qquad 21$$

Equation 19 becomes:

$$\ln \frac{P_H E}{P_E H} = \left(\ln \frac{ck}{d} + \varepsilon_1\right) + ((c-2) + \varepsilon_2) \ln H + ((2-d) + \varepsilon_3) \ln E + \ln w \qquad 22$$

or

$$\ln \frac{P_H E}{P_E H} = \ln \frac{ck}{d} + (c-2) \ln H + (2-d) \ln E + (\varepsilon_1 + \varepsilon_2 \ln H + \varepsilon_3 \ln E + \ln w) \qquad 23$$

In Equations 20-23, the terms g_0, g_1, g_2, g_3, are random variables with mean zero and finite variance; g_2 and g_3 are then truncated random variables, subject to the theoretical constraints on c and d.

Ordinary least squares applied to Equation 21 or two stage least squares applied to Equation 23 yield estimates of the mean value of the parameter distributions. The estimates are not asymptotically efficient because of the heteroskedasticity inherent in random coefficients regression, but a generalized least squares approach would require _a priori_ knowledge of the variances of the parameters about their means.

The estimation of the mean values of a distribution of parameters implies our benefit estimates are for some "mean" household. The interpretation of such a measure is best viewed asymptotically; consistent parameter estimates would imply that our estimate of benefits based on the estimate of mean parameters is a consistent measure of benefits. However, the distribution of benefits is truncated and this introduces additional problems for our benefit measure, given the random coefficients framework. (This returns us to a topic touched on briefly at the end of Chapter VI.)

The potential biases introduced by the random coefficients model can be highlighted by two extreme cases. First, consider a constrained price subsidy program which places participants without regard to their tastes (indeed, each bundle offered in the program is viewed as being held by people with the whole gamut of tastes) but such that a person with the average function parameters, if given an unconstrained cash grant equal to the dollar value of the subsidy, would buy the same bundle as he is given under the subsidy. Such a program would be said to be "perfectly efficient" for a person of average preferences in the sense that the equivalent measure of benefits equals the dollar value of the subsidy; every "subsidy dollar" becomes a "benefit dollar".

If we use the average utility function parameters to estimate benefits in this case, every household will seem to share in the perfect efficiency of the program. In fact, however, only families with the average utility function parameters will evaluate the program so highly, all other households will actually value the program <u>less</u> than the estimated measure indicates.

Thus, in this instance we would over-estimate benefits; other things equal, the larger the variance of the actual marginal rates of substitution about their mean, the larger the bias in estimating benefits.

Now consider a second case. An omniscient project director gives each participant a bundle which is perfectly efficient given the tenants true tastes. However, in calculating benefits, we use the mean parameter values for all households. In this case, we will <u>under</u>estimate benefits for all tenants except those with average preferences. Participants who are given extra housing because of their relatively stronger preference for housing, will be judged as receiving more than the efficient amount of housing when the mean parameters are used to measure benefits; thus estimated benefits will be below true benefits. Participants who are given relatively more of other commodities because of their relatively weaker preference for housing, will be judged as receiving less than the efficient amount of housing when the mean parameters are used in their case; thus estimated benefits will be below true benefits.

Thus, in this instance we would underestimate true benefits. Moreover, as in the previous case, the larger the variance about the mean value of the marginal rate of substitution, the larger the bias in estimating benefits other things being equal.

These cases can also be seen if we write the error in estimating the benefits to the i^{th} household, u_i, as:

$$u_i = B_i - B_i^*$$

where B_i is the true benefits to the i^{th} household and B_i^* is the estimated benefit.

If S_i is the subsidy to the i^{th} household then B_i and B_i^* are both necessarily less than or equal to S_i. If B_i equals S_i for all i, then u_i must be greater than or equal to zero for all i; if B_i^* equals S_i for all i, then u_i must be less than or equal to zero for all i. It is clear that the strict inequalities hold unless B_i equals B_i^* for all i. The equality of B_i and B_i^* for all i implies that all households are alike and the variance of the marginal rate of substitution about its mean is zero; that is, both biases disappear when the random coefficients framework collapses to the ordinary least squares case. (This presumes no stochastic element other than that due to the variation of parameters; this assumption allows us to isolate the consequences inherent in our random coefficients framework. Random coefficients stochastic terms differ from ordinary error terms in that we do not expect their impact to disappear as the sample grows. Ordinary parameter estimates converge in probability (hopefully) to the true parameter values, yielding consistent if not unbiased estimates of benefits even in the extreme cases (so long as all households are alike); but even in the infinite sample case, application of the (consistent) estimate of the mean parameter values does not yield consistent benefit estimates (in the extreme cases) so long as the mean values are not the correct values for every household).

Is there anything to be learned from this? Clearly, the problems are due to the existence of upper bounds for B_i and B_i^*; hopefully, the problems could be alleviated if B_i and B_i^* are sufficiently less than their maximum values. Consider the following case.

We assume a constrained housing price subsidy in which all households have the same income and receive the same bundle of goods, and the distribution of parameters is such that every household would purchase less of the subsidized commodity if given an equivalent cash grant to spend at market prices. Clearly, households with a stronger than average preference for housing will be closer to an efficient bundle than households with average preferences for housing; conversely, households with weaker than average preferences for housing will be further from an efficient bundle than households with average tastes. But this implies that households with average preferences obtain smaller benefits than the former group and larger benefits than the latter; this introduces the possibility that the benefits accruing to the average household may be the average benefits. Whether or not this attractive possibility occurs will depend on the distribution of parameters, the particular utility function in question, and also the bundle of goods and services and their cost. The complexities of this issue are a topic in themselves and we will not try to solve them. For our purposes we assume that the desired condition is approximated so long as the subsidized quantity of housing is "sufficiently" far from the quantity of housing which would be bought under an equivalent cash grant program by households with mean parameter values. However, there are some general points regarding the distribution of the u_i defined in equation 5 which we feel can be made here.

First, the truncation of the parameter distribution which we assume in the last example is not necessarily the most likely case. It is quite possible that some households will have such strong preferences for housing that they would in fact buy even more housing if given an equivalent cash grant. And if this is so, and the amount more that they would buy is large enough, then these households would receive smaller, not larger, benefits than households with average parameters. However, unless the variance of the parameters about their mean is large this factor will probably be inconsequential as the subsidized quantity becomes larger relative to the quantity which would be bought with an equivalent cash grant.

Second, if our population is that of an actual subsidy program, the distribution of parameters will be bounded such that all tenants receive positive benefits. These upper and lower bounds will vary from program to program. However, we believe it unlikely that these "tails" of the distributions are of great import; we believe that the bulk of all populations will be concentrated densely about the mean values. It is a more open question, in our minds, whether the mean values themselves alter systematically across populations.

Our main conclusion then, is that the severity of the estimation bias in the measurement of benefits is most severe when B_i or B_i^* is close to its maximum for all i, and less severe as they both fall below their maximum values. We will now present an indicator for the potential presence of these problems.

Denote H_i^s the quantity of housing obtained by the ith household under a housing price subsidy and H_i^* the quantity of housing which the ith household would consume if given an equivalent cash grant on the assumption that the average parameters of the utility function are applied in allocating the cash grant income.

B_i^* will equal S_i only if H_i^s equals H_i^*. Thus if B_i^* equals S_i for all i, it follows that

$$E_i((H_i^s - H_i^*) / H_i^s) = 0 \qquad 24$$

Moreover, if B_i^* is close to S_i for all i, the left hand side of Equation 24 will tend to be small, and the variance of the differences about this mean will also be small.

Now consider the case where H^s is such as to be efficient for all participants, given their true utility function parameters. For those cases where the marginal rate of substitution is overestimated at the subsidized bundle, H_i^* will be to the right of H_i^s. For instances in which the marginal rate of substitution is underestimated at the subsidized bundle H_i^* will be to the left of H_i^s. These results follow directly from the slope of the indifference curve being equal to the slope of any equivalent cash grant budget line (since H_i^s is efficient for all i).

In this circumstance H_i^* will be less than H_i^s for some i and greater for others. Thus the differences in the left hand side of Equation 24 will tend to offset one another. Therefore, the left hand side of Equation 24 will again tend to be small, but now the variance of the differences about their mean will be relatively large (depending on the variance of the parameters about their mean).

Thus, the larger is the left hand side of Equation 24, the less likely a severe bias due to B_i and B_i^* having upper bounds. (The left hand side of Equation 24 being small does not imply these biases, it is only a necessary condition. To see this, simply consider switching the positions of the households whose H_i^s are greater than H_i^* and those whose H_i^s are less than H_i^*; the positions are no longer efficient, and the bias "disappears", but the left hand side of Equation 24 is unchanged).

Fortunately, in our study, the left hand side of Equation 24 is generally significantly different from zero, and we therefore discount the bias as insignificant.

In assuming symmetry of the u_i, we are being heroic. However, we have at least made the problems obvious, which is in itself an advance over the blythe measurement of Marshallian consumers surplus.

A second statistical technique we use is necessitated by the theoretical constraints on the parameters of the utility functions. We anticipated before beginning our actual runs that the primitiveness of our data might make it difficult to fit the generalized CES without violating prior constraints; this proved to be true, for at least some family compositions. The Cobb-Douglas, in which the parameter estimated only represents an estimate of the average budget share spent on housing, poses no such problems.

To provide a superior estimator we implement a quadratic programming technique presented by Takayama and Judge (23). This approach allows us to impose inequality constraints on the parameters to be estimated while still using a (restricted) least squares criterion for choosing estimates.

The primary task in this formulation is to translate the problem so that all parameters to be estimated are greater than zero, <u>a priori</u>. Thus, parameters which theory tells us are always negative or which fall between a negative and a positive number, or which are greater than some negative number, all necessitate translating the matrices involved.

The transformation of the problem is determined as follows (23). If:

$$(1) \quad y = XB + u$$

$$(2) \quad r^s \leq B \leq r^u$$

then let:

$$C = B - r^s \quad \text{where}$$

$$0 \leq C \leq r^{u-r^u}$$

The problem is then to minimize

$$C' \; (X'Xr^s - X'Y) + \tfrac{1}{2} \, C'X'XC$$

Subject to:

$$C \,\_\, r^u - r^s$$

$$C \,\_\, 0$$

The realization that the distribution of parameters is of the discrete-continuous sort found in quadratic programming should act as a very strong caveat to those who would wish to interpret our results in the usual statistical framework. Assumptions of normalcy, as contained in our non-quadratic formulation, can only be roughest approximations and may even be misleading. Takayama and Judge (23) are careful to point out the tenuousness of the properties our estimates will have and while the more usual regression framework is still used where possible because of its familiarity, we hasten to warn the reader to be skeptical.

To summarize, we are in a random coefficients regression framework; we use an approximate generalized least squares technique to deal with heteroskedasticity, instrumental variables to deal with correlation between independent variables and error terms, and a quadratic programming technique to account for prior inequality constraints on parameters. The quadratic programming approach takes no account of the heteroskedasticity. We take no measures to account for errors in variables.

To the extent our assumptions are valid, our estimates are only inconsistent to the extent that our price and income variables are subject to error. This is not an ideal situation, but it seems to us to be the best presently available. The subsequent chapters present our results and an interpretation of their implications.

CHAPTER IX: THE NUMBERS

This chapter contains the bulk of our numerical results. In each case we provide an explanation of the sources and techniques used to obtain the numbers. In the next chapter we utilize the numbers to formulate tentative answers to our research questions. Hopefully, this division of results and their interpretation will allow a greater clarity than would be possible if the explanatory passages were cluttered with detailed tables.

In Chapter VII we explained our prices are based on studies by the Bureau of Labor Statistics (BLS). In forming a price of housing, utilities were excluded. This is based on our judgement that most private units do not include utilities in rent. Table 1 contains the 1971 costs of the standard BLS bundles of housing services and other commodities for the seven cities in our sample, as well as the price ratios for these cities; PH, PE, and PR indicate the costs of the housing bundle, the costs of the other commodities bundle, and the price ratio, respectively.

Four estimates of the market value of public housing units are used in this study. Three of these estimates are based on the data in Table 2. Public authorities are often required to provide to the Department of Housing and Urban

Development estimates of the local market value of public housing units by number of bedrooms. Table 2 contains such estimates for our seven cities (all values are translated to 1971 prices using the usual BLS indices). Zeroes indicate that no estimate was given. It is not known how these estimates were generated.

Each city's data are used (along with the BLS price indices) to generate estimates for all other cities. This yields seven sets of forty-two estimates each. From these we select the minimum and maximum estimates for each apartment size in each city; we also take an average of all estimates (correcting for the missing elements in Table 2). Tables 3 through 5 contain the resultant estimates of the market value of public housing. Implicit in this combinatorial approach is the assumption that public housing is everywhere the same; the use of upper and lower bounds affords some protection from errors in this assumption.

The fourth estimate of the market value of public housing is based on the rent versus housing characteristics regression equation estimated by Olsen (18). Some adaptation was needed since not all of the data required were available.

The summary characteristics of the eighty-six housing projects in the study are used to estimate the average floor on which a tenant lives, whether or not an elevator is available, the number of families in the unit and the

Table 1. 1971 Housing Prices, Other Prices, and Relative Prices, by City

	PH	PE	PR
Austin	943	5419	.1740
Boston	1404	6421	.2187
Honolulu	1728	7262	.2380
Indianapolis	1224	6007	.2038
Minneapolis-St. Paul	1210	6023	.2009
Pittsburgh	1047	6031	.1736
Washington, D.C.	1366	6134	.2227

Table 2. Local Estimates of Market Value of Public Housing Units by Number of Bedrooms and City

	0	1	2	3	4	5
Austin	0	106	125	155	0	0
Boston	93	104	114	123	134	144
Honolulu	124	147	181	237	301	0
Indianapolis	87	92	98	128	141	141
Minneapolis-St. Paul	96	108	150	182	215	228
Pittsburgh	0	155	169	204	289	0
Washington, D.C.	106	133	160	187	0	0

Table 3. Minimum Estimates of Market Value of Public Housing Units by City and Number of Bedrooms

	0	1	2	3	4	5
Austin	62	70	76	83	90	97
Boston	93	104	112	123	134	144
Honolulu	114	128	138	151	165	177
Indianapolis	81	91	98	107	117	126
Minneapolis-St. Paul	80	90	97	106	115	124
Pittsburgh	69	78	84	92	100	107
Washington, D.C.	90	101	109	120	130	140

Table 4. Maximum Estimates of Market Value of Public Housing Units by City and Number of Bedrooms[1]

	0	1	2	3	4	5
Austin	95	140	152	184	260	260
Boston	111	208	227	274	388	388
Honolulu	137	256	279	337	477	477
Indianapolis	97	181	198	238	338	338
Minneapolis-St. Paul	96	179	195	236	334	334
Pittsburgh	83	155	169	204	289	289
Washington, D.C.	108	202	220	266	377	377

[1]The figures for four and five bedroom units are the same because the maximum values for 4 bedroom units exceeded the maximum values for 5 bedroom units; we use the 4 bedroom figures for both. The problem arises because only three cities gave estimates for five bedroom unit.

Table 5. Mean Estimates of Market Value of Public Housing Units by City and Number of Bedrooms

city #BR	0	1	2	3	4	5
Austin	69	92	108	132	158	128
Boston	102	137	161	196	235	189
Honolulu	126	168	198	241	290	233
Indianapolis	90	119	140	171	206	165
Minneapolis-St. Paul	88	118	138	169	203	163
Pittsburgh	77	102	120	146	175	142
Washington, D.C.	99	133	156	191	228	184

Table 6. Summary of Cobb-Doubles Regressions

$$P_H H = (\frac{a}{b})/(1 + \frac{a}{b})\ Y + u$$

Family Composition	Estimated Coefficient	t-Statistic	No. of Observations
FS10	.334	43.85	514
FS20	.296	26.29	140
FS21	.320	26.24	164
FS31	.314	13.87	70
FS32	.343	26.98	149
FS42	.261	11.36	49
FS43	.324	21,87	119
FS54	.323	20.36	107
FS65	.283	22.54	76

like. The market value of a tenant's unit is then dependent on the city and project in which he lives and the number of bedrooms in his unit. This estimate only approximates the true Olsen measure, but it provides an attractive check on the other three measures. (The usual BLS index is used to translate estimates from 1968 New York prices to 1971 prices in other cities.)

Olsen's regression contained variables for number of bedrooms, condition, period of construction, floor (with or without elevator), burrough, number of other units, and state of furnishing. We assumed all public housing units are standard, in an "average" burrough (average the burrough variables), unfurnished, and have an elevator if more than three stories tall. The number of stories is obtained by dividing the number of units by eight, and for a given unit, the number of stories is the average number for the project. Period of construction, number of bedrooms, and number of other units are all known. Projects with over one hundred units were assumed to be sets of twelve story high rises.

The limitations of this approach are obvious; the close fit of Olsen and mean measures of value supports the hypothesis that the errors tend to cancel.

The Cobb-Douglas can be estimated without reference to the BLS data. Market rent is simply regressed against income, as explained in Chapter VI. Table 6 contains the product of nine Cobb-Douglas regressions, one for each family composition in the study. Family compositions are denoted by two numbers (the first, the number of persons in the family; the second, the number of minors in the family) prefixed by the letters FS. Thus FS43 indicates a four person household with three minors.

The generalized CES requires a distinction between quantities and prices. As we have explained in Chapter VII, quantities are measured in units of the BLS standard bundle, and the prices of housing per unit are the prices in Table 1. Division of market rent by the "price of housing" yields the quantity of housing. Table 7 contains the summary statistics and parameter estimates for the nine unconstrained generalized CES regressions. The figures in parentheses are t-statistics.

The estimates for FS20 and FS21 imply negative C's; this is not permissable. For these cases we constrained the estimate of (C-2) to be greater than or equal to -1.9. The estimates for FS32 and FS42 prove to violate the convexity conditions within the relevant range of H and E. For these cases we constrain the estimate of (2-d) to be 1.01 or greater. The results of these constrained estimations are to be found in Table 8. Since the properties of estimators such as these are unknown, we exclude summary statistics.

Tables 9 and 10, respectively, present the estimated Cobb-Douglas and generalized CES utility functions for each family composition. These functions are used as part of an iterative computer program for approximating public housing

Table 7. Summary of Unconstrained Generalized CES Regressions

$$\ln (P_H E/P_E H) = \ln ({}^{kc}/d) + (c-2)\ln H + (2-d)\ln E + \ln u$$

Family Comp.	$\ln({}^{kc}/d)$	(c-2)	(2-d)	No. of Obs.
FS10	-1.61 (-46)	-1.44 (-28)	1.11 (53)	514
FS20	-1.89 (-5.69)	-2.80 (-1.87)	1.38 (3.82)	140
FS21	-2.29 (-4.50)	-3.22 (-1.99)	1.13 (5.63)	164
FS31	-1.87 (-5.45)	-1.81 (-2.14	.904 (7.77)	70
FS32	-1.62 (-59)	-1.06 (-7.82)	.98 (31)	149
FS42	-1.79 (-17)	-1.43 (-6.1)	.87 (16)	49
FS43	-1.58 (-48)	-.83 (-10.3)	.94 (49)	119
FS54	-1.62 (-57.6)	-.965 (-18.4)	.919 (37)	107
FS65	-1.6 (-28)	-1.4 (-6.1)	1.03 (12.5)	76

Table 8. Constrained Generalized CES Regressions

$$\ln (P_H E/P_E H) = \ln (kc/d) + (c-2)\ln H + (2-d)\ln E + \ln u$$

Family Composition	Constraint	ln(kc/d)	(c-2)	(2-d)
FS20	(c-2) = -1.9	-1.74	-1.9	1.19
FS21	(c-2) = -1.9	-1.94	-1.9	1.02
FS32	(2-d) = 1.01	-1.56	-1.001	1.01
FS42	(2-d) = 1.01	-1.54	-1.006	1.01

Table 9. Cobb-Douglas Utility Functions by Family Composition

Family Comp.	Utility Function
FS10	$U = H^{.334} \times E^{.666}$
FS20	$U = H^{.296} \times E^{.704}$
FS21	$U = H^{.32} \times E^{.68}$
FS31	$U = H^{.314} \times E^{.686}$
FS32	$U = H^{.343} \times E^{.657}$
FS42	$U = H^{.261} \times E^{.739}$
FS43	$U = H^{.324} \times E^{.676}$
FS54	$U = H^{.323} \times E^{.677}$
FS65	$U = H^{.283} \times E^{.717}$

Table 10. Generalized CES Utility Functions by Family Composition

Family Comp.	Utility Function
FS10	$U = (.03H^{.6} + E^{.9})^{.8}$
FS20	$U = (1.45H^{.1} + E^{.8})^{.8}$
FS21	$U = (1.5H^{.1} + E^{.99})^{.6}$
FS31	$U = (.82H^{.2} + E^{1.1})^{.8}$
FS32	$U = (.21H^{.999} + E^{.99})^{.4}$
FS42	$U = (.21H^{.994} + E^{.99})^{.4}$
FS43	$U = (.19H^{1.17} + E^{1.06})^{.3}$
FS54	$U = (.2H^{1.03} + E^{1.08})^{.3}$
FS65	$U = (.33H^{.6} + E^{.97})^{.8}$

tenant benefits from public housing, section 236 housing, and section 236 housing with rent supplement.

Four separate sets of estimates of public housing tenant benefits from public housing are developed using the four estimates of the market value of public housing. Tables 11 through 14 present, in order, the estimated benefits, by family composition, using the Olsen, mean, minimum, and maximum estimates of the market value of public housing. Each table contains benefit estimates based on the Cobb-Douglas and generalized CES utility functions, the subsidy from government on the assumption that the cost of public housing is equal to its market value, and the subsidy from government based on the more acceptable assumption that government cost exceeds market value (here we assume cost is one hundred and twenty per cent of market value; this figure is based on recent unpublished research by the Department of Housing and Urban Development (24). Each table gives a sample mean, based on the 1388 observations in this study.

It also proves helpful to have all benefits reduced to constant dollars. The base selected is 1971 Washington, D.C. prices. It is these benefits which are used in analyzing the distribution of benefits. Table 15 through 18 present constant dollar benefits for the four measures of the market value of public housing. Also included in this table is the standard deviation of the benefits about the family composition means.

Table 11. Nominal Benefits from Public Housing - Olsen Measure

Family Comp.	Benefits/mo. C - D	Benefits/mo. Gen. CES	Subsidy/mo. Cost=1.2 Val.	Subsidy/mo. Cost = Val.
FS10	48.19	27.62	74.19	55.28
FS20	68.69	68.63	112.79	85.29
FS21	61.68	63.99	113.46	84.44
FS31	97.56	88.63	152.47	117.08
FS32	85.93	92.63	135.77	103.56
FS42	94.90	107.70	165.20	126.71
FS43	104.54	82.12	166.91	128.80
FS54	126.82	125.38	193.58	151.03
FS65	123.66	133.23	204.68	159.43
Sample Mean	75.00	66.85	120.68	91.86

Table 12. Nominal Benefits from Public Housing - Minimum Measure

Family Comp.	Benefits C - D	Benefits Gen. CES	Subsidy Cost=1.2 Val.	Subsidy Cost=Val.
FS10	51.47	27.88	77.06	57.68
FS20	44.80	46.95	72.22	51.49
FS21	32.95	37.24	61.56	41.19
FS31	47.43	48.76	76.00	53.36
FS32	40.94	35.99	65.71	45.17
FS42	41.34	33.34	71.00	48.20
FS43	45.30	11.33	71.68	49.44
FS54	52.37	34.25	78.08	54.78
FS65	48.90	49.87	75.91	52.12
Sample Mean	46.22	33.26	72.73	51.67

Table 13. Nominal Benefits from Public Housing - Maximum Measure

Family Comp.	Benefits/mo. C - D	Benefits/mo. Gen. CES	Subsidy/mo. Cost=1.2 Val.	Subsidy/mo. Cost = Val.
FS10	82.34	31.85	132.49	103.87
FS20	109.13	103.81	195.40	154.13
FS21	94.63	93.22	184.66	143.77
FS31	130.65	113.73	215.41	169.53
FS32	115.52	138.15	192.27	150.64
FS42	121.02	153.96	223.93	175.65
FS43	131.86	125.51	222.97	175.51
FS54	154.44	168.76	248.80	197.05
FS65	153.93	171.79	275.63	218.55
Sample Mean	107.49	92.35	183.23	143.99

Table 14. Nominal Benefits from Public Housing - Mean Measure

Family Comp.	Benefits/mo. C - D	Benefits/mo. Gen. CES	Subsidy/mo. Cost=1.2 Val.	Subsidy/mo. Cost = Val.
FS10	64.63	29.48	97.98	75.11
FS20	71.08	70.79	115.62	87.65
FS21	59.79	62.61	108.45	80.27
FS31	87.49	80.83	134.82	102.37
FS32	76.54	78.99	119.00	89.58
FS42	82.49	87.51	139.62	105.39
FS43	90.05	62.02	140.35	106.66
FS54	108.11	98.80	159.96	123.01
FS65	104.70	110.29	165.85	127.07
Sample Mean	75.42	55.21	118.60	90.14

Table 15. Constant Dollar Benefits from Public Housing - 1971 Wash., D.C. Prices
Olsen Measure

Family Comp.	Benefits/mo. C - D	Stnd. Dev. C - D	Benefits/mo. Gen. CES	Stnd. Dev. Gen. CES
FS10	46.14	19.44	27.62	12.03
FS20	66.26	20.03	66.26	18.87
FS21	61.68	29.35	63.99	27.43
FS31	90.28	32.60	82.63	22.63
FS32	86.48	24.36	92.86	33.80
FS42	126.63	28.91	101.33	47.38
FS43	103.30	25.70	79.63	47.47
FS54	125.79	21.27	123.77	39.55
FS65	123.24	20.93	132.75	24.57

Table 16. Constant Dollar Benefits from Public Housing - 1971 Wash., D.C. Prices
Minimum Measure

Family Comp.	Benefits C - D	Stnd. Dev. C - D	Benefits Gen. CES	Stnd. Dev. Gen. CES
FS10	48.70	15.24	27.88	12.06
FS20	42.96	16.47	45.08	16.80
FS21	32.95	24.37	37.24	25.43
FS31	43.13	28.10	44.76	22.37
FS32	41.02	25.49	35.72	29.47
FS42	38.21	22.91	29.42	36.03
FS43	44.42	23.43	9.90	38.56
FS54	51.70	23.14	33.16	25.27
FS65	48.49	21.44	49.48	22.17

Table 17. Constant Dollar Benefits from Public Housing - 1971 Wash., D.C. Prices Maximum Measure

Family Comp.	Benefits/mo. C - D	Stnd. Dev. C - D	Benefits/mo. Gen. CES	Stnd. Dev. Gen. CES
FS10	78.72	24.64	57.68	12.02
FS20	105.51	22.84	100.48	16.12
FS21	94.63	29.76	93.22	25.08
FS31	121.38	37.05	106.53	21.79
FS32	116.30	24.16	138.67	33.81
FS42	115.78	29.08	145.93	50.78
FS43	130.50	26.57	122.35	52.66
FS54	153.21	22.54	166.80	42.56
FS65	153.44	27.31	171.17	31.26

Table 18. Constant Dollar Benefits from Public Housing - 1971 Wash., D.C. Prices
Mean Measure

Family Comp.	Benefits/mo. C - D	Stnd. Dev. C - D	Benefits/mo. Gen. CES	Stnd. Dev. Gen. CES
FS10	61.45	15.72	29.48	11.99
FS20	68.53	17.19	68.31	16.18
FS21	59.79	25.95	62.61	25.52
FS31	80.75	31.20	75.20	22.25
FS32	76.94	23.60	79.03	31.35
FS42	78.29	24.68	81.82	42.47
FS43	88.90	23.51	59.81	44.04
FS54	107.24	20.93	97.29	38.05
FS65	104.29	19.38	109.83	21.69

To estimate the benefits public housing tenants would receive from Section 236 housing and Section 236 with Rent Supplement, we must decide how much housing the tenants would be given and what rents they would pay under the two programs. To determine rents requires knowing the amount which would cover costs on the given apartment presuming a one per cent mortgage (this is the minimum rent in Section 236 housing).

These figures were estimated by averaging the quantity of housing given to each family, household composition by household composition, in a sample of 1,205 actual Section 236 housing tenants, and by averaging the fraction of the market value of the occupied unit paid as rent by all observations in the sample which pay in excess of twenty-five per cent of their income in rent (these families are paying the minimum allowable rent), again, family composition by family composition. Table 19 presents the estimated number of BLS standard housing units given to each family composition, and the estimated fraction of market value paid as rent by same.

Thus, public housing tenants of each family composition are assigned a quantity of housing services, based on the amount received by their Section 236 counterparts, and are expected to pay as rent at least the smallest rent allowed these same counterparts. With the rent supplement provision, a ceiling rent equal to 25 per cent of income is also imposed.

Table 19. Housing Granted and Minimum Rent Paid in Section 236 Housing

Family Composition	Amount of Housing	Minimum Rent (as fraction of value)
FS10	1.77	.63
FS20	1.82	.65
FS21	1.84	.66
FS31	2.00	.68
FS32	2.06	.66
FS42	2.10	.66
FS43	2.15	.67
FS54	2.13	.69
FS65	2.39	.60

In Section 236 housing, according to recent unpublished research by the Department of Housing and Urban Development (24), the market value of units is approximately ninety-one per cent of the full cost rent of the unit (where full cost includes maintenance costs). Tables 20 and 21 present the estimated nominal benefits to public housing tenants if they were to shift to Section 236 housing and Section 236 housing with Rent Supplement as outlined above. Table 22 presents constant dollar benefits from Section 236 housing with Rent Supplement; again, the base is 1971 Washington, D.C. prices and the standard deviations are included.

Table 23 lists the differences, expressed as a faction of pre-program housing, between the amount of housing obtained in public housing, H^s, and the amount which would be obtained under an equivalent cash grant, H^*, for each of the four estimates of the value of public housing, using the Cobb-Douglas utility function. Table 24 does the same but for the generalized CES utility function. Table 25 contains this information for Section 236 with Rent Supplement using both utility functions.

Table 26 lists the increase in housing consumed by public housing tenants expressed as a fraction (in excess) of original consumption, for all four measures of the market value of public housing. This table also lists the change in the consumption of other commodities expressed as

Table 20. Nominal Benefits to Public Housing Tenants If Put into Section 236 Housing

Family Comp.	Benefits/mo. C - D	Subsidy/mo.
FS10	-51.89	98.09
FS20	-18.82	92.47
FS21	-27.65	88.02
FS31	-32.24	99.24
FS32	-25.50	96.62
FS42	-18.26	105.68
FS43	-25.00	103.14
FS54	-36.04	102.68
FS65	8.04	130.60
Sample Aug	-41.41	96.45

Table 21. Nominal Benefits to Public Housing Tenants If Put into Sec. 236 with Rent Supp.

Family Comp.	Benefits/mo. C - D	Benefits/mo. Gen. CES	Subsidy/mo.
FS10	111.76	29.77	184.37
FS20	98.22	95.04	167.53
FS21	100.94	97.29	166.41
FS31	119.68	104.08	186.68
FS32	120.15	148.08	183.04
FS42	99.14	135.56	181.02
FS43	124.96	122.14	194.75
FS54	134.73	151.33	203.81
FS65	123.50	143.25	207.82
Sample Aug.	113.40	87.90	181.08

Table 22. Constant Dollar Benefits to Public Housing Tenants from Section 236 with Rent Supplement - 1971 Wash., D.C. Prices

Family Comp.	Benefits C - D	Stnd. Dev. C - D	Benefits Gen. CES	Stnd. Dev. Gen. CES
FS10	106.65	48.28	29.10	10.86
FS20	95.37	47.22	92.36	14.67
FS21	100.94	46.35	97.29	18.07
FS31	112.44	47.08	98.46	24.03
FS32	120.92	47.32	148.71	30.00
FS42	94.85	49.25	128.27	42.28
FS43	123.61	45.03	119.07	47.07
FS54	133.73	48.71	149.62	35.91
FS65	123.06	47.38	142.79	19.75

Table 23. DIF = $(H^s - H^*)/H^m$ for Public Housing and the Cobb-Douglas Utility Function

Family Comp.	DIF (Olsen)	DIF (Min.)	DIF (Max.)	DIF (Mean)
FS10	.33	.35	.96	.56
FS20	.74	.22	1.60	.76
FS21	.80	.33	1.55	.77
FS31	.68	.07	1.26	.54
FS32	.71	.13	1.22	.56
FS42	.96	.15	1.49	.72
FS43	.86	.11	1.34	.65
FS54	1.00	.06	1.28	.72
FS65	1.21	.16	1.82	.87

Table 24. DIF = $(H^s - H^*)/H^m$ for Public Housing and the Generalized CES Utility Function

Family Comp.	DIF (Olsen Meas.)	DIF (Min. Meas.)	DIF (Max. Meas.)	DIF (Mean Meas.)
FS10	.55	.59	1.40	.88
FS20	.83	.43	1.93	.96
FS21	1.00	.40	1.91	.97
FS31	1.23	.40	1.97	1.06
FS32	.11	-.30	.40	.03
FS42	-.23	-.83	.14	-.41
FS43	-1.81	-1.92	-1.93	-1.83
FS54	.86	-.37	1.39	.53
FS65	1.60	.29	2.32	1.19

Table 25. DIF = $(H^s - H^*)/H^m$ for Section 236 with Rent Supplement

Family Comp.	DIF Cobb-Douglas	DIF. Gen. CES
FS10	1.95	2.56
FS20	1.69	1.97
FS21	1.51	1.89
FS31	1.42	2.07
FS32	1.40	.60
FS42	1.61	.27
FS43	1.42	-1.92
FS54	1.48	1.36
FS65	1.65	2.03

Table 26. HCHNG = $(H^s-H^m)/H^m$ and ECHNG = $(E^s-E^m)/E^m$ for Public Housing

Family Comp.	HCHNG (Olsen)	HCHNG (Min)	HCHNG (Max)	HCHNG (Mean)	ECHNG
FS10	.67	.71	1.53	1.01	.018
FS20	1.10	.58	2.14	1.13	.013
FS21	1.12	.52	2.04	1.09	.011
FS31	1.32	.51	2.05	1.15	.018
FS32	1.20	.41	1.86	1.02	.016
FS42	1.46	.47	2.08	1.18	.013
FS43	1.42	.42	2.03	1.15	.014
FS54	1.74	.50	2.33	1.38	.013
FS65	1.74	.44	2.46	1.33	.010
Sample Mean	1.09	.57	1.89	1.10	.014

a fraction (in excess) of original consumption. Table 27 contains this information for Section 236 housing with Rent Supplement.

The final group of numbers of particular interest are the regressions involving the constant dollar benefits of Tables 15 through 18 and Table 22 and the attributes of the recipients. These investigations uncover the distribution of real benefits among the participants.

Table 28 contains the estimated coefficients of income, race, and age (the latter two being represented by dummy variables), from a regression involving only these variables as independent variables and the benefit measures from Tables 15 through 18. The age variable distinguished heads of household below 62 years of age (value, zero) from heads 62 years of age or older (value, one). The race variable distinguished non-whites (value, zero) from whites. Summary statistics are included.

Table 29 contains the same data as Table 28 for a regression in which family composition dummies were included as independent variables. Table 30 is similar to Table 29 but for the addition of city of residence dummies.

Tables 31 through 33 are analogous to Tables 28 through 30 but present results using the constant dollar benefits from Table 22 as the dependent variables.

Table 27. HCHNG = $(H^s-H^m)/H^m$ and ECHNG = $(E^s-E^m)/E^m$ for Section 236 Housing with Rent Supplement

Family Comp.	HCHNG	ECHNG
FS10	2.69	.015
FS20	2.18	.010
FS21	2.02	.014
FS31	2.15	.016
FS32	2.04	.015
FS42	2.12	.008
FS43	2.06	.012
FS54	2.25	.009
FS65	2.17	.006
Sample Aug	2.32	.013

Table 28. Distribution of Public Housing Benefits by Income (Y), Age (62+), and Race (W)[1]

Mkt. Value	Util. Function	Y	62+	W	F	R^2
Olsen	C - D	.012 (.0008)	-24.7 (1.9)	22.2 (7.1)	195	.30
Olsen	Gen. CES	.006 (.001)	-39.5 (2.3)	36.8 (8.7)	157	.25
Minimum	C - D	.007 (.0005)	.91 (1.2)	8.7 (4.5)	74	.14
Minimum	Gen. CES	-.007 (.0007)	-5.9 (1.45)	21.5 (5.5)	49	.10
Maximum	C - D	.016 (.0008)	-14.2 (1.8)	18.8 (6.9)	218	.32
Maximum	Gen. CES	.012 (.0013)	-56.7 (2.8)	44.2 (10.6)	247	.35
Mean	C - D	.005 (.0006)	-8.7 (1.5)	12.0 (5.6)	48	.09
Mean	Gen. CES	.002 (.0009)	-29.1 (2.11)	30.2 (7.5)	99	.18

[1]Figures in parentheses are standard errors.

Table 29. Distribution of Public Housing Benefits by Income (Y), Age (62+), and Race (W) - Family Composition Included[1]

Mkt. Value	Util. Function	Y	62+	W	F	R^2
Olsen	C - D	-.002	6.0	12.6	204	.62
		(.00007)	(2.0)	(5.3)		
Olsen	Gen. CES	-.014	6.6	16.4	280	.69
		(.0008)	(2.1)	(5.7)		
Minimum	C - D	-.011	7.5	6.9	49	.28
		(.0006)	(1.5)	(4.2)		
Minimum	Gen. CES	-.014	6.7	10.8	74	.37
		(.0007)	(1.7)	(4.7)		
Maximum	C - D	.006	9.1	11.5	135	.52
		(.0008)	(2.2)	(5.9)		
Maximum	Gen. CES	-.014	7.11	11.6	491	.80
		(.0009)	(2.2)	(6.0)		
Mean	C - D	-.003	8.3	8.3	78	.38
		(.0007)	(1.7)	(4.7)		
Mean	Gen. CES	-.014	6.99	11.0	200	.62
		(.0007)	(1.9)	(5.2)		

[1]Numbers in parentheses are standard errors.

Table 30. Distribution of Public Housing Benefits by Income (Y), Age (62+), and Race (W) - Family Composition and City Included[1]

Mkt. Value	Util. Function	Y	62+	W	F	R^2
Olsen	C - D	.011 (.0006)	3.3 (1.7)	8.3 (4.6)	207	.72
Olsen	Gen. CES	-.011 (.0007)	3.9 (1.84)	11.51 (4.91)	274	.77
Minimum	C - D	-.008 (.0004)	4.3 (1.0)	3.6 (2.75)	180	.69
Minimum	Gen. CES	-.011 (.0005)	4.4 (1.3)	6.82 (3.7)	131	.62
Maximum	C - D	.010 (.0008)	5.3 (1.9)	7.8 (5.1)	144	.64
Maximum	Gen. CES	-.011 (.0008)	4.30 (1.99)	6.61 (5.3)	449	.85
Mean	C - D	.008 (.0005)	4.6 (1.3)	4.8 (3.5)	154	.66
Mean	Gen. CES	-.011 (.0006)	4.3 (1.6)	6.47 (4.3)	232	.74

[1]Figures in parentheses are standard errors.

Table 31. Distribution of Tenant Benefits from Section 236 Housing with Rent Supplement by Income (Y), Age (62+), and Race[1]

Util. Function	Y	62+	W	F	R^2
C - D	.004 (.0005)	-4.8 (1.0)	-4.9 (3.9)	58	.11
Gen. CES	.003 (.001)	-64.4 (2.5)	41.9 (9.6)	280	.38

[1]Figures in parentheses are standard errors.

Table 32. Distribution of Tenant Benefits from Section 236 Housing with Rent Supplement by Income (Y), Age (62+), and Race - Family Composition Included[1]

Util. Function	Y	62+	W	F	R^2
C - D	.004 (.0005)	2.3 (1.2)	-2.9 (3.2)	88	.41
Gen. CES	-.02 (.0006)	1.64 (1.55)	5.91 (4.17)	966	.89

[1]Figures in parentheses are standard errors.

Table 33. Distribution of Tenant Benefits from Section 236 Housing with Rent Supplement by Income (Y), Age (62+), and Race - Family Composition and City Included[1]

Util. Function	Y	62+	W	F	R^2
C - D	.004 (.0004)	1.7 (1.1)	-7.6 (2.8)	100	.55
Gen. CES	-.020 (.006)	2.0 (1.5)	.58 (3.9)	731	.90

[1]Figures in parentheses are standard errors.

CHAPTER X: CONCLUSIONS

It is the academician's mark to ignore the aphorism about simple answers for simple questions. The basic questions in this study are simple ones: what are the direct benefits to public housing tenants from several alternative programs; how are these benefits distributed. The answers to these questions are not simple, and even more sadly, they are not entirely clear cut. However, in this chapter we attempt to formulate tentative answers to the questions based on the machinations of the last nine chapters. First we will present our results, then we will point out some of their limitations.

Choice of a best estimate of tenant benefits from public housing could be a problem since we have four different measures of the market value of subsidized units and two indifference maps. Fortunately, the two most attractive measures, the Olsen measure and the mean measure display a remarkable degree of agreement. (The minimum and maximum measures are only intended for use as upper and lower bounds on the market value and are, therefore, not so attractive as the other two measures.)

The use of two different utility functions highlights the speculativeness of work such as this. The comparison of public housing with the other programs is not greatly affected by the change in utility functions; however, the actual benefit estimates are definitely affected. The generalized CES function implies benefits 11 to 28 per cent lower than the Cobb-Douglas for the sample averages.

The 28 per cent drop in sample benefits from Section 236 with rent supplement in going from the Cobb-Douglas to the generalized CES starkly shows the risks involved in presuming aggregate elasticities are applicable to persons of low income. If the Cobb-Douglas is the correct functional form in analyzing one person households, then Section 236 housing with Rent Supplement yields, on average, \$0.62 in tenant benefits for every dollar spent by society, while if the generalized CES is a better specification, benefits average only \$0.15 on the dollar!

The estimates for the parameters of the generalized CES lead to the rejection of the hypothesis that the true functional form is the Cobb-Douglas for every family composition save FS32 and FS42. However the latter are both cases in which the inequality constraints were imposed. Since we expect the statistical distortions introduced by the constraints are probably most severe in the neighborhood of the boundaries, we cannot place much surety in the failure to reject the Cobb-Douglas hypothesis in these cases.

More significant is the result that the Cobb-Douglas function does not seem a useful approximation of other forms.

It seems likely that public housing tenants would prefer to be in Section 236 housing with Rent Supplement than in public housing. There is almost no doubt, on the other hand, that public housing tenants would suffer greatly if transferred to Section 236 housing <u>without</u> rent supplement. Table 20 records a $41.41 <u>loss</u> to public housing tenants from Section 236 housing as a sample average. The losses would be even greater under the generalized CES utility function.

Some households would have to pay rent in excess of their income in Section 236 housing. We arbitrarily assigned the losses in these cases to be equal to the household's income. Since such a scheme would invite starvation, we feel such an assignment of loss not to be excessive. Of course, the averages mask the fact that some households would gain from a move to Section 236 housing; however, the point remains that Section 236 housing cannot serve by itself as a suitable substitute for public housing.

Section 236 housing so clearly fails as a substitute for public housing that we do not give it any more attention. However, one significant point arises from a comparison of Section 236 housing with Section 236 with Rent Supplement.

Note that in Table 10 the sample average benefits rise from -41 to 113 dollars while the subsidy rises from 96 to 181 dollars. This means that 85 dollars of rent supplement subsidy yield 144 dollars in benefits!

Rent supplements are constrained cash grants which would seem never to be preferred, by the recipient, to an equal (or greater) unconstrained cash grant. Yet in the move from Section 236 housing to Section 236 with Rent Supplement, the supplements seem equivalent to an unconstrained cash grant greater than themselves. To understand what is happening here we have to realize that the recipient is now moving from an unconstrained position to a constrained one, but from one constrained position to another.

In Section 236 housing, the tenant is given more housing than he himself would buy with the subsidy; however, as he is given additional constrained cash grants in the form of rent supplements (The constraint is artificial; the tenant is forbidden to buy more housing, a commodity he already has relatively too much of and does not care to buy more of until he first gets more of other goods.), the previous constraint becomes less bothersome; at a new, higher income, the excessive amount of housing he receives from the program does not seem like so much too much, and the <u>previous</u> dollars of subsidy bring him more benefit than previously.

The point is that marginal benefits may exceed the marginal subsidy which generated them. Consider a household which will always try to spend one quarter of its income on housing. If government provides such an individual with one dollar's worth of housing and three dollar's worth of other goods, the benefits equal the subsidy, four dollars. If government gives the person only one dollar's worth of housing, the benefits fall short of the subsidy; now an added three dollars of other goods will raise benefits from below one dollar to four dollars, an increase in excess of the increased subsidy!

The chief significance of this is in viewing the effective tax rates associated with some subsidies. If the rent paid by tenants rises $0.25 for every additional dollar of income, the typical response is to view the effective tax rate as 25 per cent. In fact, this may be, and probably is, incorrect. The income of a subsidy recipient should be the equivalent income he receives under the subsidy, as this is the best measure of his real income. It would seem that programs which impose relatively too much of a commodity will display marginal benefits from income supplements in excess of the marginal income and (since average benefits converge to a maximum) the marginal benefits are declining. If a one dollar increase in income increases benefits by one and a half dollars, the last quarter will increase benefits by more than a quarter but by less than thirty seven and a half cents. Thus the effective tax rate is less, and perhaps significantly less than 25 per cent. (Programs imposing too little of a commodity would then display effective tax rates in excess of the usual rate.)

We now shift our attention to the comparison of public housing to Section 236 housing with Rent Supplement.

The comparison of public housing with Section 236 with Rent Supplement requires looking to both benefits and costs. It may be the case that one program is preferred by the tenants, but if this entails excessive costs society may not be willing to underwrite the scheme.

Estimated benefits from public housing exceed those from Section 236 with Rent Supplement in only one case, that in which both the maximum measure of public housing units' market value and the generalized CES utility function are assumed. If we assume the normality of the distribution of average benefits across the population and apply a t test, the difference is significant at the .005 per cent level.

In all other cases, Section 236 housing with Rent Supplement is found to be more beneficial to tenants than public housing, and the differences are significant at the .005 per cent level (again subject to the normality assumption about the distribution of benefits). Since our *a priori* expectation is that the maximum measure of the market value of public housing units is a considerable overestimate of the true market value, and since this a priori expectation is reinforced by the comparison of the three measures based on agency data with the Olsen measure, we feel secure in accepting the hypothesis that Section 236 housing with Rent Supplement would, on the whole, be preferred by public housing tenants if they were given the opportunity to choose.

We have already pointed out, with respect to Section 236 housing, that despite the average results, some individual tenants might be made better off by a change different from that preferred on average. This is a suitable place to note that our approach assumes cardinal character for utility and the comparability of individual utilities. However it is <u>not</u> the case that our approach must treat equally a dollar of subsidy to all persons. By analyzing the distribution of benefits by family characteristics, we can judge whether the distribution of benefits is skewed according to some socially preferred pattern. Thus the examination of benefits will not be complete until we analyze the distribution of benefits under the two schemes. However, we postpone this part of the investigation until after a discussion of the cost side of the subsidy coin.

The ratio of benefits to subsidies is important in terms of both efficiency and equity. Between programs yielding equal benefits, that with the higher benefit-subsidy ratio should be chosen as it is more efficient, unless society's tastes demand the imposition of housing vis-a-vis other goods. There is also a trade-off when a higher benefit per household program has a lower benefit-subsidy ratio; the program with the lower benefits per household but higher benefit subsidy ratio can yield greater aggregate benefits at equal cost to the other program, but the benefits would be spread over more households. With the single exception of the maximum measure of market value coupled with the Cobb-Douglas utility function, public housing is always estimated to be capable of generating more benefits at equal cost vis-a-vis Section 236 with Rent Supplement. Of course there are less benefits per household, but when even the massive public housing program only reaches some 7 per cent of those eligible for it (21) equity concerns begin to demand that the cream be thinned some.

The differences in benefit-subsidy ratios are slight, (highest and lowest are only 5 per cent apart) under the Cobb-Douglas specification. In this instance, the choice between public housing and Section 236 with Rent Supplement would seem to rest on the strength of our desire to do significant good for some as opposed to some good for many. However, under the generalized CES specification, the spread of ratios is much greater (with Section 236 with Rent Supplement having an efficiency rating only 66 per cent that of public housing under the Olsen measure). Here significant benefits to some come at a significantly greater cost, and we expect society would be unwilling to substitute such a structure for public housing.

In summary Section 236 with Rent Supplement would likely be preferred by public housing tenants over their present positions. It is most likely tenant benefits would rise from 29 to 87 per cent. On the other hand, the subsidy involved would most likely rise by 70 to 96 per cent. (These ranges are based on the Olsen and mean measures of the market value of public housing units.) We feel it unlikely that policy makers would be willing to allow such an added concentration of resources among a relatively small subset of the needy.

There still remains the issue of the distribution of benefits among participants. For this analysis we turn our attention from the nominal benefits to benefits measured in constant dollars (1971 Washington, D.C. prices are our base). Nominal benefits are suitable for judging the actual cost to government of programs, and are sufficiently accurate to reflect the average level of benefits. However we presume that a variation of real benefits among locations is a possible drawback of a program, (we would not want participants to have to suffer because of regional cost differentials), and thus real benefits, or constant dollar benefits, are the measure we subject to analysis here.

In addition to the presumption that real benefits ought not vary with geographical location, there are four other equity concerns which we believe underly the rationale for subsidy programs. First, persons with lower incomes should tend to receive larger benefits as they are in greater need; second, benefits should not vary with race; third, elderly households should receive greater benefits as they hold a position of distinction in our society; and fourth, larger families should receive more housing than smaller ones (and perhaps should receive larger benefits since there are more persons, usually children, in such families).

To assure that families with higher incomes receive lower benefits, other things equal, programs generally correlate income and subsidized rent. Public housing projects have rent schedules which impose higher rents on higher income families. (Regression of subsidized rents against income and a constant indicate the rate of increase of rents with income is small, on the order of one per cent; the regression has an R^2 of .52 and an F of 1478.) Section 236 with Rent Supplement generally increases rent by 25 per cent of any rise in income. However, the lesson learned earlier about the relation between marginal benefits and marginal subsidy applies here as well. Families of identical tastes may receive different benefits from identical subsidies if their incomes differ. In a program which imposes relatively too much of one good, families with higher incomes may obtain higher benefits. A decline in subsidy may not overcome the increased benefits associated with an increase in income. Thus we must look to the actual distribution of benefits to see if a given program is equitable; the distribution of subsidies is not sufficient information.

The simple regression of benefits against income, (Table 28), a dummy variable for race, and another dummy to distinguish elderly (62 years of age or older) heads of households from others, clearly indicates that public housing benefits depend positively on income, whites receive more benefits than non-whites, and the elderly receive greater benefits than others. The only exceptions are for the minimum measure of market value benefits generated with a generalized CES utility function; in this case benefits vary inversely with income, and the elderly receive fewer benefits than others. In all instances, coefficients are significant at the .005 per cent level.

The income coefficient is not really misbehaved if one wants larger families to receive larger benefits than smaller families. Family size and income are strongly correlated and if dummy variables for family composition are introduced (Table 29), the sign of the income term becomes significantly negative in every case except for the maximum measure of market value benefits generated with a Cobb-Douglas utility function. Larger families clearly receive significantly larger benefits than smaller families. The sign of the age variable is significantly positive in every case for this regression; the elderly are concentrated in one and two person households which receive relatively fewer benefits, explaining the change in sign as family composition is included or deleted. The race variable continues to indicate whites receive significantly more benefits.

The introduction of geographic location into the regression to accompany income, age, race, and family composition leads to interesting results (Table 30). First, location is a significant factor in the distribution of benefits. Comparison indicates that benefits tend to be higher in cities with higher relative prices of housing, as one might expect. Also, while the signs of the age and race parameters are unchanged, and larger families still appear to receive larger benefits, the income parameter becomes significantly positive again in all of the Cobb-Douglas cases except that of the minimum measure of market value. This would seem to indicate that cities with lower incomes tend to provide higher benefits to public housing tenants. Such an end could be achieved by such cities requiring lower rents of all tenants than do cities with higher incomes, or by providing larger subsidies. We tested the hypothesis that cities with lower incomes tend to have lower subsidized rents, apart from income, and rejected it (based on a Spearman rank correlation coefficient of zero).

The uniform difference in sign between the income coefficients for the Cobb-Douglas specifications and those for the generalized CES specifications makes the point one more time that simplistic approaches to the issues of tenant benefits, such as the reliance on aggregate demand parameters for information, is fraught with problems. The Cobb-Douglas specification would support the belief that the public housing programming is basically inequitable but for institutional corrections made at the city level (and even then people of relatively high incomes who happen to live in cities of generally lower incomes receive windfall gains while people of relatively low incomes who happen to live in cities of generally higher incomes receive windfall losses). However, the generalized CES specification would tend to refute such a view, and support the notion that geographical discrepancies are simple inequities, not correctives.

In passing we note with some surprise that the R^2's in Tables 29 and 30 indicate that the distribution of average benefits is remarkably systematic.

The results for benefits from Section 236 housing with Rent Supplement (Tables 31-33) follow much the same pattern as public housing benefits except for the race coefficient which is negative under the Cobb-Douglas specification and positive under the generalized CES specification. (This variable is probably irrelevant since we assigned tenants to Section 236 units without regard to race in our conceptual experiment while in a true to life program, discrimination would be likely to appear, if at all, through program administration.) The age coefficient is negative when city and family composition are omitted, and turns positive when they are included. The income coefficient is positive when city and family composition are omitted, is negative for the generalized CES, but positive for the Cobb-Douglas when family composition is included, and is negative for the generalized CES specification and positive for the Cobb-Douglas specification when location is added to the regression. The difference between the Cobb-Douglas benefits income coefficient and the generalized CES benefits income coefficient in this last case is the same as in the analogous public housing regressions, and leads us to suspect that marginal benefits have a significantly different relation to marginal income for the generalized CES utility function than for the Cobb-Douglas.

In summary, we conclude that the equity criteria regarding income, age, and family size, are met, although the reason for the negative correlation between income and benefits is not clear. The race and locational equity concerns, are not met by the public housing programs, the locational criterion is not met by Section 236 housing with Rent Supplement, and we are not able to judge whether Section 236 with Rent Supplement would meet the race criterion.

The last element in analyzing the programs is to look at the extent to which consumers patterns of consumption are altered by participation in the programs. Presumably one goal of transfers in kind is to limit the benefits to changes in the purchase of the subsidized commodity. Tables 26 and 27 indicate that this goal is successfully achieved; consumption of non-housing goods increases by less than two per cent for both public housing and Section 236 housing with Rent Supplement. On the other hand, consumption of housing by public housing tenants is most likely 187 to 205 per cent, on average, of what it was prior to participation, and under Section 236 with Rent Supplement consumption would most likely be 347 per cent of pre-participation consumption. These figures indicate that both programs radically change the consumption patterns of participants.

This concludes our presentation of answers to the two basic questions of this study. Before passing on to the limitations of our results, we will present the income elasticities of housing demand implied for our sample by the estimates of the generalized CES utility function.

The aggregate income elasticity for a group is a weighted averages of the individual elasticities where the weights are individual shares of aggregate income. For our sample, the aggregate income elasticity with respect to nominal income is .42. If we use De Leeuw's (9) estimate that aggregate income elasticity with respect to permanent income is one hundred and twenty per cent of the nominal income elasticity, we get an estimate of .5. This figure does not coincide with the popular wisdom and indicates that analysts and government agencies should reconsider before using aggregate elasticity estimates of unity in studying the behavior of low income groups.

We now turn our attention to the limitations of our results. Foremost are problems connected with the choice and estimation of utility functions.

The discrepancies in quantitative results between the Cobb-Douglas specification and the generalized CES are sufficient to make us wonder what would be the consequence of using still another functional form. It would seem that extensive work on the pattern of consumer's tastes is needed before work of this sort can be carried very far.

The Cobb-Douglas specification presents few estimation problems; any body of data containing all positive budget shares will lead to consistent results. The cost of this virtue is high; with price and income elasticities fixed at one, the data is allowed to convey very little information. Consequently, without strong *a priori* confidence in the unitary elasticity hypothesis, the Cobb-Douglas results must be viewed with skepticism.

The generalized CES specification leaves the data much more freedom. Unfortunately, this freedom is much abused as a number of the family compositions violate the theoretical parameter constraints.

At first sight, utilization of theoretical restrictions to obtain acceptable parameter estimates seems to be simple massaging of the data. However the matter is not so simple. If the variance of a parameter estimator is likely to be large, as it is in our framework, there is a definite probability, even if the specified utility function is the true function, that the parameter estimate fall outside of the allowed region. Imposition of the inequality constraints is effectively the acceptance of the proposition that if a constraint is violated, the most likely true parameter value (given the validity of the specification itself) is a boundary value.

At some point, violation of the parameter constraints becomes sufficient evidence to reject the hypothesis that the specification is the true one. For example, we estimated a VES utility function which violated its constraints for nearly every family composition. We accepted the universality of the violations as sufficient evidence that the VES was an improper specification. Thus, in practice, we have settled on a middle ground between the use of theoretical constraints as a test of the model and as an intrinsic part of the model.

We are wary of our results because the constraints had to be imposed, but we do not think we should reject our conclusions on the basis of this problem.

Finally, we refer to Tables 23 to 25 as evidence that the bias which arises as the subsidized quantity, H^s of housing approaches the equivalent cash grant quantity of housing, H^*, is not going to be strongly felt in our study. The average distance between H^* and H^s is a substantial fraction of the initial bundle of housing in all cases. None of the average differences are near zero.

CHAPTER XI: FURTHER NOTIONS

While this study seemed to us at times to take forever, it is actually a very limited effort. In this chapter we try to suggest extensions which would improve on our efforts. We feel secure that our general format has fruitful applications in many areas, and that despite our shortcomings, we have taken a step in the right direction (even if our only success is to illuminate how much is obscured in other, "approximate" methods, we have served a worthwhile purpose).

We see three areas in which our study could be most easily improved. First, the data base could be better. Second, the model could be extended to more than two commodities. Third, additional specifications of the utility function could be fitted.

The easiest improvement in the data base would come from surveys which could verify the rent and income observations. There are incentives in the public housing program to understate income, and this may or may not extend to the previous rent reports. Verification would minimize this source of biased errors in variables.

More significant than improvements in income and market rent data, finer estimates of the prices of housing and other goods would be difficult to achieve. Within the confines of our present framework, in which housing services are presumed to be an homogeneous good, the only likely alternative would be to specify a particular set of characteristics (preferably characteristics common in low income housing) and do a survey of housing in the neighborhood of public housing projects to develop price data for each housing project. A similar survey would have to be done for other goods, although it might be sufficient to use the BLS city data for the price of non-housing goods.

Other gains could be scored if the two-commodity world were extended to allow for more alternatives. To break the non-housing commodity into several components would allow relaxation of the assumption of fixed relative prices within the non-housing set and would also present opportunities to compare non-housing transfers, such as food stamps, with housing programs.

Subdivision of the housing commodity would bring improvements of another sort, and new problems as well. Recent work by Olsen (18) and Grether and Mieszkowski (14) has attempted to estimate the determinants of the market rent or value of housing units. The common form is to regress market rent (or value) against a series of housing characteristics; the coefficients of the independent variables may be viewed as market prices. The regression equation for a single city would be used in conjunction with BLS inter-city indices, or separate regressions could be compiled for each city. A new estimation problem which arises here is that some of the independent variables are dummy variables; to include dummy variables in the utility function introduces new problems, but they are not especially difficult to resolve.

If one or more variables in the utility function only take on specific integer values, the utility maximization problem becomes an exercise in integer programming (so long as there are no "holes" in the series of possible integer values). Fortunately, the least squares estimation problem can be transformed, in much the same fashion as we utilize here to account for inequality constraints in least squares estimation, to an integer programming framework.

Thus, subdivision of the housing commodity would be one avenue to improving the price data, if we are willing to abandon our present confines in which housing services are treated as a homogeneous commodity. The prices associated with independent variables in a regression with market rent (or value) as dependent variable would probably be superior to the BLS housing composite.

Another gain from additional regression estimates of housing prices would be the refinement of estimates of the market value of public housing units. The high degree of agreement between the Olsen and mean measures in this study indicates that these improvements are not likely to be very significant.

The final area of improvement which we see is the estimation and analysis of additional functional forms of the utility function. The Stone-Geary, or displaced Cobb-Douglas, would be an obvious possibility, but it would probably be fruitful to develop new functional forms with desirable properties; regularly behaved (and simple) income and price elasticities would be attractive properties. Certainly one would want to allow for a variety of elasticity patterns in selecting forms, but there is an added issue of comparability of forms.

Ramsey (22) has built on the work of Box and Cox (3) to investigate specification error detection in regression analysis. It would be most desirable to implement these techniques in choosing among alternative specifications of the utility function. Of course these tests are generally dependent on assumptions about the true distribution of errors, which indicates that further research into the distributional consequences of theoretical parameter constraints is also needed so we can judge how closely theoretical distributions may be approximated in our problem.

A part of the specification investigation would hopefully include a search for criteria by which to use violations of theoretical parameter constraints as possible but not necessary grounds for rejection of a specification.

In Chapter II we noted that households probably become more attached to a dwelling the longer they reside there.

This effect could be accounted for by treating the length of residence as a factor in the real quantity of housing services consumed. We can specify the utility function (say Cobb-Douglas) as a function of real housing services, $H^{(R)}$, and other goods consumed, E.

$$U = H^{(R)^a} E^b;$$

The marginal condition is then:

$$P_H/P_E = \frac{a}{b} \frac{E}{H^{(R)}}.$$

Now we simply specify $H^{(R)}$ to be a simple function of the physical quantity of housing, H, and the length of residence, t.

$$H^{(R)} = e^{gt} H$$

The estimation problem is then:

$$\ln \frac{P_H H}{P_E E} = \ln \frac{a}{b} - gt + \ln u$$

This approach is attractive in its simplicity as the only new data needed are the lengths in residence of households. The approach is especially attractive in problems in which one is trying to estimate the losses incurred by households dislocated by some federal program. Indeed, under the usual competitive assumptions, moving costs and a loss of "neighborhood capital", as described here, may be the only identifiable costs to relocation.

Finally, our estimates could be substantially improved if we could collect panel data, that is both times series and cross section. This data can be used to estimate the variance-covariance matrix of parameters across households. This step would then permit estimation of the distribution of actual benefits about their mean where we have only discussed the distribution of mean benefits.

The use of panel data would probably make worthwhile a much more technical investigation of the problems associated with estimating a bounded benefit measure. Such an investigation would involve a substantial contribution to the literature of consumers' surplus which has to date concerned itself only with points on the demand curve.

Finally, this study can serve simply as an example for persons interested in the direct benefits from other government consumer subsidy programs, such as Section 236 housing, the Farmer's Home Loan program, Food Stamps, and Medicare.

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I am glad I have done it. I am glad it is done. And I am looking forward to new experiences with new people, new people who will have a long way to go to match those I have mentioned here.